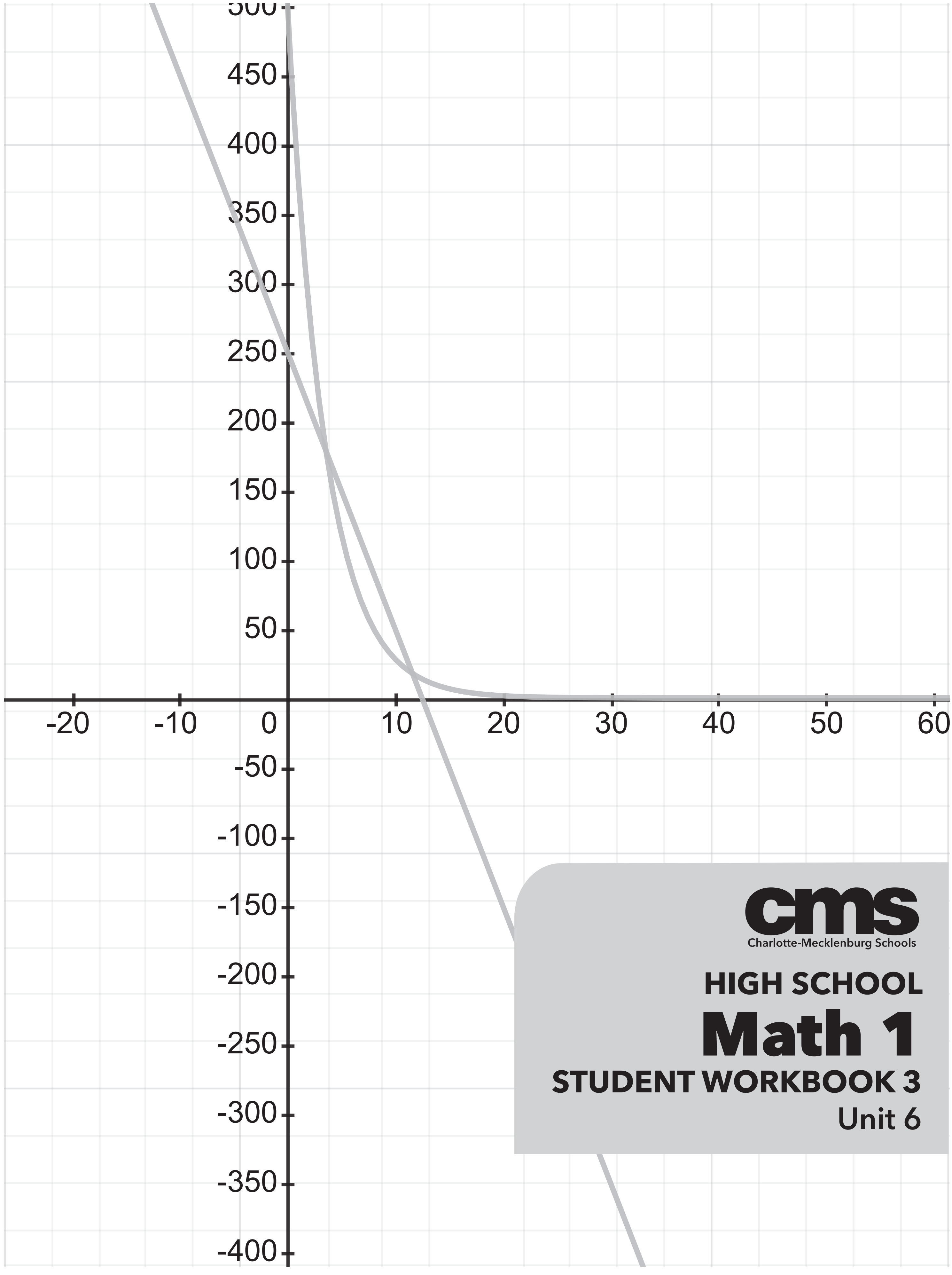

Charlotte-Mecklenburg Schools

HIGH SCHOOL

Math 1

STUDENT WORKBOOK 3

Unit 6

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Lesson 1: Properties of Exponents (Part One)

Learning Targets

- I can explain and use a rule for multiplying terms with exponents that have the same base.
- I can explain and use a rule for raising an exponential expression to a power.

Warm-up: Interpreting Exponents

The expression 3^4 means there are "4 factors of 3."

The expression can be written in the expanded form where $3^4 = 3 \cdot 3 \cdot 3 \cdot 3$.

Explain the meaning of each of the following expressions and write them in expanded form.

1. 2^4

2. x^3

3. $4^3 \cdot 5^2$

4. $x^3 y$

5. $(5x)^4$

Activity 1: Product of Powers

1. Complete the table to explore patterns in the exponents when multiplying expressions with the same base. You may skip a single box in the table, but if you do, be prepared to explain why you skipped it. The first row is done for you.

Expression	Expanded	Fewest number of exponents
$8^2 \cdot 8^4$	$(8 \cdot 8)(8 \cdot 8 \cdot 8 \cdot 8)$	8^6
a. $2^5 \cdot 2^3$		
b. $3 \cdot 3^7$		
c. $x^2 \cdot x^3$		
d.	$(m \cdot m \cdot m \cdot m \cdot m \cdot m \cdot m \cdot m \cdot m)(m \cdot m \cdot m \cdot m)$	
e. $(5y^2)(4y^3)(-y)$		
f. $(r^2s^5)(r^3s^4)$		

If you chose to skip one entry in the table, which entry did you skip? Why?

2. Use the patterns you found in the table to rewrite $x^a \cdot x^b$ as an equivalent expression with a single exponent, like $x^{\square}$.

3. Use your rule to write $x^a \cdot x^0$ with a single exponent. What does this tell you about the value of x^0?

Activity 2: Power of a Power

1. Complete the table to explore patterns in the exponents when applying a power to a power. The first row is done for you.

Expression	Expanded	Fewest number of exponents
$(3^2)^5$	$3^2 \cdot 3^2 \cdot 3^2 \cdot 3^2 \cdot 3^2 = (3 \cdot 3)(3 \cdot 3)(3 \cdot 3)(3 \cdot 3)(3 \cdot 3)$	3^{10}
a. $(2^3)^4$		
b.	$x^4 \cdot x^4 = (x \cdot x \cdot x \cdot x)(x \cdot x \cdot x \cdot x)$	
c. $(xy^2)^3$		
d. $(6n^4)^3$		
e. $(-3r^3s^4)^2$		

2. Use the patterns you found in the table to rewrite $(x^a)^b$ as an equivalent expression with a single exponent, like $x^{\square}$.

3. Jada rewrote the expression $(3x^5)^2(2x^6)$. Examine her work and decide if you agree or not.
   - If you agree, explain why each step is correct.
   - If you disagree, explain the error and provide the correct equivalent expression.

$$\begin{aligned}(3x^5)^2(2x^6) &= 3^2 \cdot x^{10} \cdot 2 \cdot x^6 \\ &= 6 \cdot 2 \cdot x^{10} \cdot x^6 \\ &= 12x^{16}\end{aligned}$$

Activity 3: More Than One Rule

Rewrite each expression using the fewest possible exponents. Show or explain your reasoning.

1. $(x^5)^2(x^3)$

2. $(6y^3)^2(y^4)$

3. $(rs^2)(r^6)^2(s)$

4. $(3m)^2(2m^4n^7)$

Lesson Debrief

Lesson 1 Summary and Glossary

Exponents are used to express repeated multiplication. For example, x^5 means there are five factors of x and can be expanded into the equivalent form of $x \cdot x \cdot x \cdot x \cdot x$.

Expressions that include exponents can be written in different ways by using the properties of exponents.

One of these properties is called the **Product of Powers Rule**. This is used when multiplying exponential expressions with the same base.

Product of Powers Rule

$$x^a \cdot x^b = x^{a+b}$$

Why it works

$$x^3 \cdot x^5 = \underbrace{(x \cdot x \cdot x)}_{\text{three factors of } x}\underbrace{(x \cdot x \cdot x \cdot x \cdot x)}_{\text{five factors of } x} = \underbrace{x^8}_{\text{eight factors of } x}$$

Another of these properties is called the **Power of a Power Rule**. This is used when applying a power to an expression that also has a power.

Power of a Power Rule

$$(x^a)^b = x^{ab}$$

Why it works

$$(x^2)^3 = \underbrace{(x \cdot x)(x \cdot x)(x \cdot x)}_{\text{three groups of two factors of } x} = \underbrace{x^6}_{\text{six factors of } x}$$

For some expressions, both properties are needed. For example, in the expression $(4x^3)^2(x^5)$:

$(4x^3)^2(x^5)$ The expression $4x^3$ is being raised to a power of 2, so multiply each exponent by 2.(The 4 has an exponent of 1 and can be written as 4^1: $(4^1x^3)^2$)

$= (4^2x^6)(x^5$ The expressions x^6 and x^5 have the same base and are being multiplied, so add the exponents. Rewrite 4^2 as 16.

$= 16x^{11}$

Product of Powers Rule: When multiplying two exponential expressions that have the same base, add the exponents: $x^a \cdot x^b = x^{a+b}$

Power of a Power Rule: When raising an exponential expression to a power, multiply the powers: $(x^a)^b = x^{ab}$

Unit 6 Lesson 1 Practice Problems

1. Rewrite the expression using the fewest possible exponents.

   a. $x^3 \cdot x^{10}$

   b. $(s^6)^5$

   c. $(4m^2)(-m^8)(m)$

   d. $(2d^5 f)^3(d^4 f^7)$

2. Find the value of each unknown exponent that makes the equation true.

   a. Find the value of b: $3y^7 \cdot 2y^b = 6y^{12}$

   b. Find the value of b: $(4x^6)^b = 64x^{18}$

   c. Find the values of b and c: $(r^9 s^3)(7r^2 s)^b = 49r^c s^5$

3. Select the **four** expressions that are equivalent to $-2x^5(4x^3)^2$

   a. $-2x^5(4x^3)(4x^3)$

   b. $(-2 \cdot 4)(x^5 \cdot x^3 \cdot x^2)$

   c. $-2x^5(16x^6)$

   d. $(-2 \cdot x \cdot x \cdot x \cdot x \cdot x)(4 \cdot x \cdot x \cdot x)(4 \cdot x \cdot x \cdot x)$

   e. $-8x^{10}$

   f. $-32x^{11}$

   g. $-512x^{11}$

4. Which expression is **equal to** $(6x^5y)^3(xy^4)$?

   a. $6x^9y^7$

   b. $6x^{16}y^7$

   c. $18x^{16}y^7$

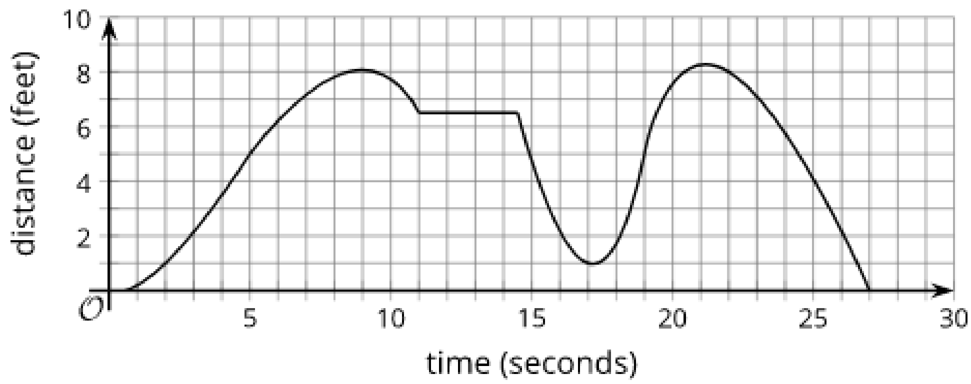
   d. $216x^{16}y^7$

5. A tennis ball is dropped from an initial height of 30 feet. It bounces five times, with each bounce height being about $\frac{2}{3}$ of the height of the previous bounce.

   Sketch a graph that models the height of the ball over time. Be sure to label the axes.

(From Unit 5)

6. This graph represents Andre's distance from his bicycle as he walks in a park.



a. For which intervals of time is the value of the function decreasing?

b. For which intervals is it increasing?

c. Describe what Andre is doing during the time when the value of the function is increasing.

(From Unit 5)

7. (*Technology required.*) A survey wanted to determine if there was a relationship between the number of joggers who used a local park for exercise and the temperature outside. The data in the table display their findings.

   Use graphing technology to create a scatter plot of the data.

Temperature in Fahrenheit, x	Number of joggers, y
15	4
30	8
30	8
41	4
42	16
49	20
49	14
55	16
66	34
72	44
85	40
94	15

   a. Is a linear model appropriate for this data? Explain your reasoning.

   b. If the data seems appropriate, create the line of best fit. Round to two decimal places.

   c. What is the slope of the line of best fit and what does it mean in this context? Is it realistic?

   d. What is the y-intercept of the line of best fit and what does it mean in this context? Is it realistic?

(From Unit 4)

8. Select **all** equations that can result from adding these two equations or subtracting one from the other.

$$\begin{cases} x+y=12 \\ 3x-5y=4 \end{cases}$$

a. $-2x-4y=8$

b. $-2x+6y=8$

c. $4x-4y=16$

d. $4x+4y=16$

e. $2x-6y=-8$

f. $5x-4y=28$

(From Unit 3)

9. Solve each system of equations without graphing.

a.
$$\begin{cases} 2x+3y=5 \\ 2x+4y=9 \end{cases}$$

b.
$$\begin{cases} \frac{2}{3}x+y=\frac{7}{3} \\ \frac{2}{3}x-y=1 \end{cases}$$

(From Unit 3)

10. Select **all** the expressions that equal $2^8 \cdot 2^4$.

a. 2^4

b. 2^{12}

c. 4^6

d. 2^{32}

e. 4^{32}

(Addressing NC.6.EE.1)

Lesson 2: Properties of Exponents (Part Two)

Learning Targets

- I can explain and use a rule for dividing exponential expressions.
- I can evaluate x^0 and explain why it makes sense.
- I know what it means if an expression is raised to a negative power.

Bridge

Name two fractions that are equivalent to $\frac{2}{5}$. Explain or show your reasoning.[1]

Warm-up: A Value of One

Solve each equation mentally.

1. $\frac{12}{x} = 1$
2. $\frac{x}{9} = 1$
3. $\frac{x}{4} \cdot 5 = 5$
4. $\frac{3 \cdot 7 \cdot 11}{3 \cdot x} = 7$

[1] Adapted from IM K–5, Grade 4, Unit 2, Lesson 7 https://curriculum.illustrativemathematics.org/K5/index.html,

Activity 1: Quotient of Powers

1. Complete the table to explore patterns in the exponents when dividing expressions with exponents. You may skip a single box in the table, but if you do, be prepared to explain why you skipped it. The first two rows are done for you.

Expression	Expanded	Fewest number of exponents
$\frac{2^{10}}{2^3}$	$\frac{2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2}{2\cdot2\cdot2} = \frac{2\cdot2\cdot2}{2\cdot2\cdot2} \cdot 2^7 = 1 \cdot 2^7$	2^7
$\frac{8x^5}{6x^3}$	$\frac{2\cdot4\cdot x\cdot x\cdot x\cdot x\cdot x}{2\cdot3\cdot x\cdot x\cdot x} = \frac{2\cdot x\cdot x\cdot x}{2\cdot x\cdot x\cdot x} \cdot \frac{4}{3} \cdot x \cdot x = 1 \cdot \frac{4}{3}x^2$	$\frac{4}{3}x^2$
a. $\frac{15x^6}{5x^2}$		
b.	$\frac{2\cdot5\cdot r\cdot r\cdot r\cdot r\cdot r}{2\cdot r\cdot r\cdot r} = \frac{2\cdot r\cdot r\cdot r}{2\cdot r\cdot r\cdot r} \cdot 5r^2 = 1 \cdot 5r^2$	
c. $\frac{-8m^4n^7}{2m^3n}$		
d. $\frac{6xy^3}{15y^2}$		

If you chose to skip one entry in the table, which entry did you skip? Why?

2. Use the patterns you found in the table to write $\frac{x^a}{x^b}$ as an equivalent expression with a single exponent, like $x^{\square}$.

3. Use your rule to write each of the following with a single exponent, like $x^{\square}$.

Expression	Expanded	Expression with a single exponent
a. $\frac{x^3}{x^3}$	$\frac{x \cdot x \cdot x}{x \cdot x \cdot x} = 1$	
b. $\frac{r^4}{r^4}$	$\frac{r \cdot r \cdot r \cdot r}{r \cdot r \cdot r \cdot r} = 1$	
c. $\frac{m^5}{m^5}$	$\frac{m \cdot m \cdot m \cdot m \cdot m}{m \cdot m \cdot m \cdot m \cdot m} = 1$	

4. What does this tell you about the value of x^0? Explain your reasoning.

Activity 2: Negative Exponents

Rewrite the expressions using the rules of exponents. The final expression should include only positive exponents.

1. $x^{-7} \cdot x^3$

2. $(2rs^2)^{-3}$

3. $\frac{8m^6}{4m^9}$

4. $(3x^2y)^{-1}(6x^3y^4)$

5. $\frac{(r^4s^{-2})^3}{rs^4}$

Lesson Debrief

Lesson 2 Summary and Glossary

The **Quotient of Powers Rule** is used when dividing expressions with exponents.

Quotient of Powers Rule

$$\frac{x^a}{x^b} = x^{a-b}$$

Why it works

$$\frac{x^5}{x^3} = \frac{x \cdot x \cdot x \cdot x \cdot x}{x \cdot x \cdot x} = \frac{x \cdot x \cdot x}{x \cdot x \cdot x} \cdot x^2 = 1 \cdot x^2 = x^{5-3}$$

three factors of x in the numerator and denominator

5-3=2 factors of x

The **Zero Exponent Rule** defines the value of an expression with a power of 0.

Zero Exponent Rule

$$x^0 = 1$$

Why it works

Using the expanded form

$$\frac{x^3}{x^3} = \frac{x \cdot x \cdot x}{x \cdot x \cdot x} = 1$$

Using the Quotient of Powers Rule

$$\frac{x^3}{x^3} = x^{3-3} = x^0$$

Since $\frac{x^3}{x^3} = 1$ and $\frac{x^3}{x^3} = x^0$ then $x^0 = 1$

The **Negative Exponent Rule** is used to write expressions with negative exponents as expressions with positive exponents.

Negative Exponent Rule	Why it works
$$x^{-a} = \frac{1}{x^a}$$	Using the expanded form $$\frac{x^3}{x^5} = \frac{x \cdot x \cdot x}{x \cdot x \cdot x \cdot x \cdot x} = \frac{x \cdot x \cdot x}{x \cdot x \cdot x} \cdot \frac{1}{x^2} = 1 \cdot \frac{1}{x^2} = \frac{1}{x^2}$$ Using the Quotient of Powers Rule $$\frac{x^3}{x^5} = x^{3-5} = x^{-2}$$ Since $\frac{x^3}{x^5} = \frac{1}{x^2}$ and $\frac{x^3}{x^5} = x^{-2}$ then $x^{-2} = \frac{1}{x^2}$

For some expressions, several properties are needed. For example, the expression $\frac{x^4y^6}{x^3y^8}$:

$\frac{x^4y^6}{x^3y^8}$	The expressions are being divided, so subtract the exponents.
$= x^{4-2}y^{6-8}$ $= xy^{-2}$	The y has a negative exponent, so rewrite it as the reciprocal with a positive exponent.
$= x \cdot \frac{1}{y^2}$ $= \frac{x}{y^2}$	Write the product as a single expression.

Quotient of Powers Rule: When dividing two exponential expressions that have the same base, subtract the exponents: $\frac{x^a}{x^b} = x^{a-b}$

Zero Exponent Rule: $x^0 = 1$

Negative Exponent Rule: $x^{-a} = \frac{1}{x^a}$

Unit 6 Lesson 2 Practice Problems

1. Rewrite each expression using the Zero Exponent Rule.
   a. r^0
   b. $5m^0$
   c. $(3xy)^0$
   d. $\frac{4rs}{s^0}$

2. Which of the following is equivalent to $\frac{12x^{10}y^{15}}{4x^5y^3}$?
   a. $3x^2y^5$
   b. $3x^5y^{12}$
   c. $8x^2y^5$
   d. $8x^5y^{12}$

3. Priya says, "I can figure out the value of 3^{-1} by looking at other powers of 3. I know that $3^2 = 9$, $3^1 = 3$, and $3^0 = 1$."
   a. What pattern do you notice?
   b. If this pattern continues, what should be the value of 3^{-1}?
   c. If this pattern continues, what should be the value of 3^{-2}?

4. Kiran used the exponent rules to rewrite the expression $\frac{(5x^2y^4)^3}{x^{-4}y^6}$. His steps are shown below. For each step, identify which property was applied.

$$\frac{(5x^2y^4)^3}{x^{-4}y^6} = \frac{125x^6y^{12}}{x^{-4}y^6}$$
$$= 125x^{10}y^6$$

5. Rewrite each expression using the fewest number of exponents.

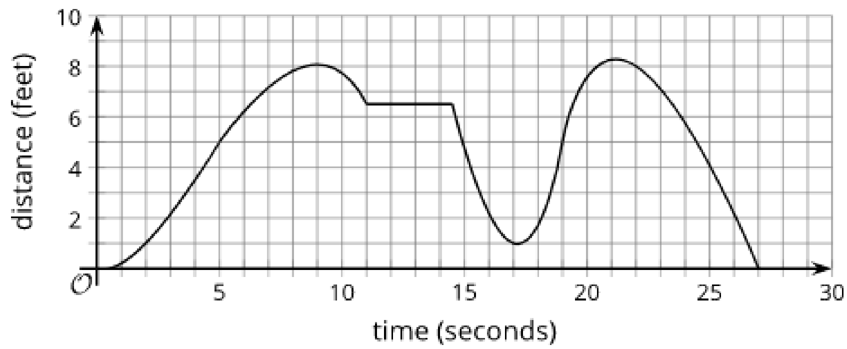
   a. $(4r^9)(3r^7)$

   b. $(5m^6n^3)^2$

   c. $(xy^2)(2x^3y^4)^3$

(From Unit 6, Lesson 1)

6. Here is the graph of function f, which represents Andre's distance from his bicycle as he walked in a park.



a. Estimate $f(5)$

b. Estimate $f(17)$

c. For what values of t does $f(t) = 8$?

d. For what values of t does $f(t) = 6.5$?

e. For what values of t does $f(t) = 10$?

(From Unit 5)

7. Two children set up a lemonade stand in their front yard. They charge $1 for every cup. They sell a total of 15 cups of lemonade. The amount of money the children earned, R dollars, is a function of the number of cups of lemonade they sold, n.

   a. Is 20 part of the domain of this function? Explain your reasoning.

   b. What does the range of this function represent?

   c. Describe the set of values in the range of R.

   d. Is the graph of this function discrete or continuous? Explain your reasoning.

(From Unit 5)

8. Select **all** systems that are equivalent to this system of equations: $\begin{cases} 4x + 5y = 1 \\ x - y = \frac{3}{8} \end{cases}$

   a. $\begin{cases} 4x + 5y = 1 \\ 4x - 4y = \frac{3}{2} \end{cases}$

   b. $\begin{cases} x + \frac{5}{4}y = \frac{1}{4} \\ x - y = \frac{3}{8} \end{cases}$

   c. $\begin{cases} 4x + 5y = 1 \\ 5x - 5y = 3 \end{cases}$

   d. $\begin{cases} 8x + 10y = 2 \\ 8x - 8y = 3 \end{cases}$

   e. $\begin{cases} x + y = \frac{1}{5} \\ x - y = \frac{3}{8} \end{cases}$

(From Unit 3)

9. A catering company is setting up for a wedding. They expect 150 people to attend. They can provide small tables that seat 6 people and large tables that seat 10 people.

   a. Find a combination of small and large tables that seats exactly 150 people.

   b. Let x represent the number of small tables and y represent the number of large tables. Write an equation to represent the relationship between x and y.

   c. Explain what the point $(20, 5)$ means in this situation.

   d. Is the point $(20, 5)$ a solution to the equation you wrote? Explain your reasoning.

(From Unit 3)

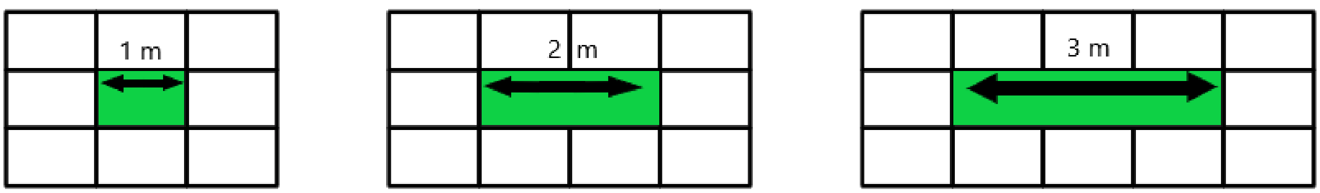
Lesson 3: Growing and Growing

Learning Targets

- I can compare growth patterns using calculations and graphs.

Bridge

A homeowner wants to build a garden with concrete tiles around the outside. He has room for the garden to vary in length but not width. He's not sure what size he wants the garden to be. Here are sketches of gardens that are 1, 2, and 3 meters long. The homeowner needs to know how many concrete tiles will be needed for different possible garden lengths.



1. Create a table to show how many tiles will be needed if the garden is 1, 2, 3, 4, or 5 meters long.

2. Describe the way the pattern is growing.

Warm-up: Two Expressions

Here are two different algebraic expressions. What do you notice or wonder?

a. $70+5+5+5+5+5$

b. $70\cdot 5\cdot 5\cdot 5\cdot 5\cdot 5$

Activity 1: A Genie in a Bottle

You are walking along a beach and your toe hits something hard. You reach down, grab onto a handle, and pull out a lamp! It is sandy. You start to brush it off with your towel. Poof! A genie appears.

He tells you, "Thank you for freeing me from that bottle! I was getting claustrophobic. You can choose one of these purses as a reward."

- Purse A, which contains $1,000 today. If you leave it alone, it will contain $1,200 tomorrow (by magic). The next day, it will have $1,400. This pattern of $200 additional dollars per day will continue.
- Purse B, which contains one penny today. Leave that penny in there, because tomorrow it will (magically) turn into two pennies. The next day, there will be four pennies. The amount in the purse will continue to double each day.

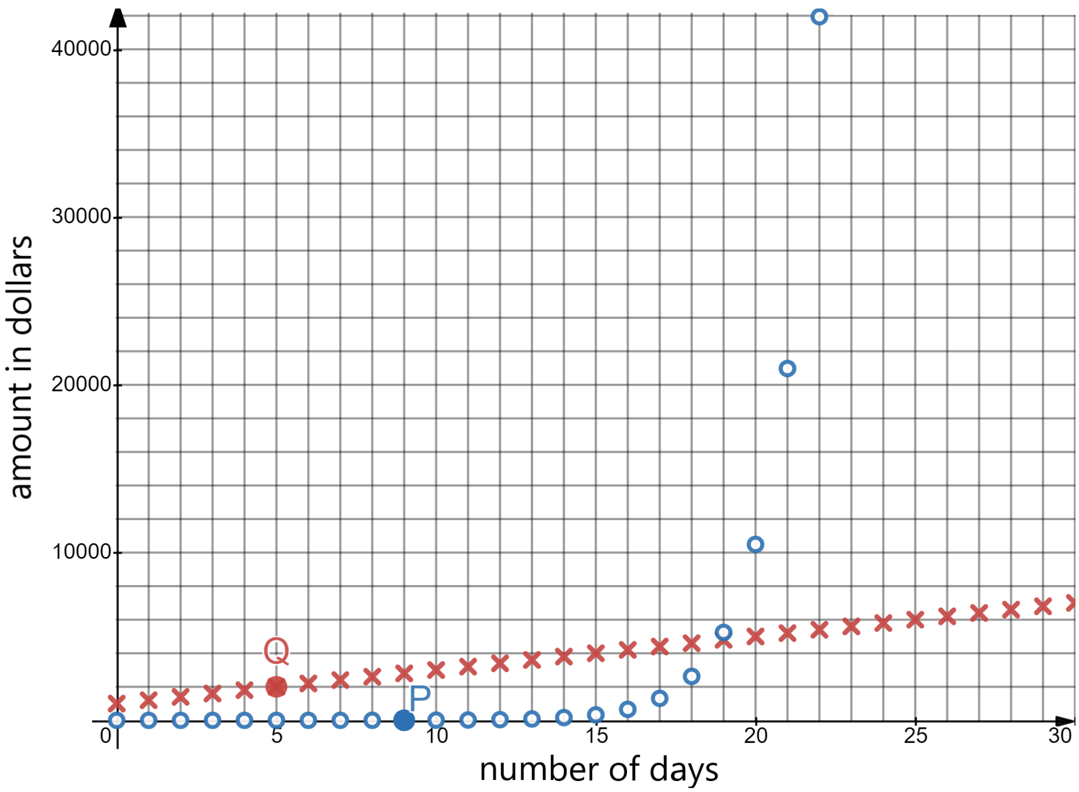
1. How much money will be in each purse after a week? After two weeks?

2. The genie later added that he will let the money in each purse grow for three weeks. How much money will be in each purse then?

3. Which purse contains more money after 30 days?

Activity 2: Graphing the Genie's Offer

Here are graphs showing how the amount of money in the purses changes. Remember purse A starts with \$1,000 and grows by \$200 each day. Purse B starts with \$0.01 and doubles each day.



1. Which graph shows the amount of money in purse A? Which graph shows the amount of money in purse B? Explain how you know.

2. Points $P(9, 5.12)$ and $Q(5, 2000)$ are labeled on the graph. Explain what they mean in terms of the genie's offer.

3. What are the coordinates of the vertical intercept for each graph? Explain how you know.

4. When does purse B become a better choice than purse A? Explain your reasoning.

5. Knowing what you know now, which purse would you choose? Explain your reasoning.

Are You Ready For More?

“Okay, okay,” the genie smiles, disappointed; “I will give you an even *more* enticing deal.” He explains that purse B stays the same, but purse A now increases by $250,000 every day. Which purse should you choose?

Lesson Debrief

Lesson 3 Summary and Glossary

When we repeatedly double a positive number, it eventually becomes *very* large. Let's start with 0.001. The table shows what happens when we begin to double:

0.001	0.002	0.004	0.008	0.016

If we want to continue this process, it is convenient to use an exponent. For example, the last entry in the table, 0.016, comes from 0.001 being doubled 4 times, or $(0.001) \cdot 2 \cdot 2 \cdot 2 \cdot 2$, which can be expressed as $(0.001) \cdot 2^4$.

Even though we started with a very small number, 0.001, we don't have to double it that many times to reach a very large number. For example, if we double it 30 times, represented by $(0.001) \cdot 2^{30}$, the result is greater than $1,000,000$.

Throughout this unit, we will look at many situations where quantities grow or decrease by applying the same factor repeatedly.

Unit 6 Lesson 3 Practice Problems

1. Which expression equals 2^7?

   a. $2+2+2+2+2+2+2$

   b. $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

   c. $2 \cdot 7$

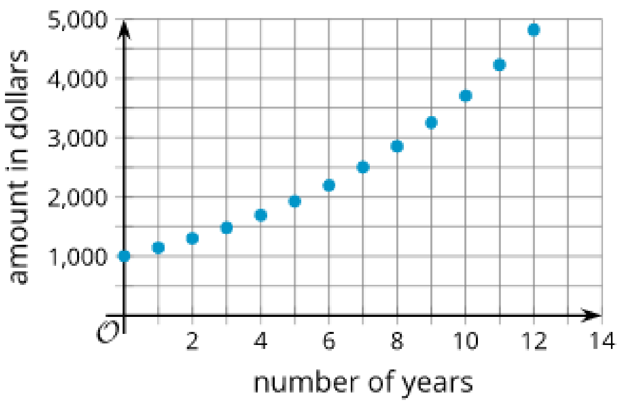
   d. $2+7$

2. Evaluate the expression $3 \cdot 5^x$ when x is 2.

3. The graph shows the yearly balance, in dollars, in an investment account.

   a. What is the initial balance in the account?

   b. Is the account growing by the same number of dollars each year? Explain how you know.



   c. A second investment account starts with \$2,000 and grows by \$150 each year. Sketch the values of this account on the graph.

   d. How does the growth in the two account balances compare?

4. Jada rewrites $5 \cdot 3^x$ as $15x$. Do you agree with Jada that these are equivalent expressions? Explain your reasoning.

5. Investment account 1 starts with a balance of $200 and doubles every year. Investment account 2 starts with $1,000 and increases by $100 each year.

    a. How long does it take for each account to double?

    b. How long does it take for each account to double again?

    c. How does the growth in these two accounts compare? Explain your reasoning.

6. Write the following expression with only negative exponents: $(x^5 y^{-2} x^8)^3$

(From Unit 6, Lesson 2)

7. Lin says that a snack machine is like a function because it outputs an item for each code input. Explain why Lin is correct.

(From Unit 5)

8. At a gas station, a gallon of gasoline costs $3.50. The relationship between the dollar cost of gasoline and the number of gallons purchased can be described with a function.

   a. Identify the input variable and the output variable in this function.

   b. Describe the function with a sentence of the form "_____ is a function of _____."

   c. Identify an input-output pair of the function and explain its meaning in this situation.

(From Unit 5)

9. Noah and Lin are solving this system: $\begin{cases} 8x + 15y = 58 \\ 12x - 9y = 150 \end{cases}$

Noah multiplies the first equation by 12 and the second equation by 8, which gives: $\begin{cases} 96x + 180y = 696 \\ 96x - 72y = 1,200 \end{cases}$

Lin says, "I know you can eliminate x by doing that and then subtracting the second equation from the first, but I can use smaller numbers. Instead of what you did, try multiplying the first equation by 6 and the second equation by 4."

a. Do you agree with Lin that her approach also works? Explain your reasoning.

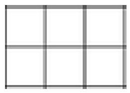
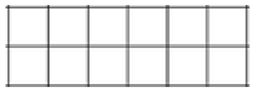
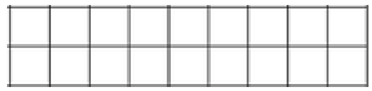
b. What are the smallest whole-number factors by which you can multiply the equations in order to eliminate x?

(From Unit 3)

10. Solve this system of linear equations without graphing: $\begin{cases} 7x + 11y = -2 \\ 7x + 3y = 30 \end{cases}$

(From Unit 3)

11. Each small square below represents 1 square unit.

Figure 1	**Figure 2**	**Figure 3**	**Figure 4**	**Figure 5**
				

a. Indicate the number of squares in figures 1, 2, 3, 4, and 5.

b. Describe how the pattern is growing.

c. Write an equation representing the number of boxes, y, in figure x.

(Addressing NC.6.EE.9)

Lesson 4: Patterns of Growth

Learning Targets

- I can use words and expressions to describe patterns in tables of values.
- When I have descriptions of different types of growth relationships between two quantities, I can write expressions and create tables of values to represent them.

Bridge

Tyler and Mai purchased gas from the same gas station. Tyler purchased 4 gallons for a total cost of $14. Mai purchased 12 gallons for a total cost of $42. What is the price per gallon?

Warm-up: Tables of Values

Which one doesn't belong? Explain your reasoning.

Table A

x	1	2	3	4	8
y	8	16	24	32	64

Table B

x	0	2	4	6	8
y	0	16	32	48	64

Table C

x	0	1	2	3	4
y	1	4	16	64	256

Table D

x	0	1	2	3	4
y	4	8	12	16	20

Activity 1: Growing Stores

A food company currently has five convenience stores. It is considering two plans for expanding its chain of stores.

1. Plan A: Open 20 new stores each year.

   a. Use technology to complete a table for the number of stores for the next 10 years, as shown here.

Year	Number of stores	Difference from previous year
0	5	
1	25	
2		
3		
4		
5		
6		

   b. What do you notice about the difference from year to year?

   c. If there are n stores one year, how many stores will there be a year later?

2. Plan B: Double the number of stores each year.

   a. Use technology to complete a table for the number of stores for the next 10 years under each plan, as shown here.

Year	Number of stores	Difference from previous year	Factor from previous year
0	5		
1			
2			
3			
4			
5			
6			

   b. What do you notice about the difference from year to year?

   c. What do you notice about the factor from year to year?

   d. If there are n stores one year, how many stores will there be a year later?

Are You Ready For More?

Suppose the food company decides it would like to grow from the five stores it has now so that 5 years from now, it will have at least 600 stores but no more than 800 stores.

1. Come up with a plan for the company to achieve this where it adds the same number of stores each year.

2. Come up with a plan for the company to achieve this where the number of stores increases by the same factor each year. (Note that you might need to round the outcome to the nearest whole store.)

Activity 2: Flow and Followers

Here are verbal descriptions of two situations, followed by tables and expressions that could help to answer one of the questions in the situations.

- Situation 1: A person has 80 followers on social media. The number of followers triples each year. How many followers will she have after 4 years?
- Situation 2: A tank contains 80 gallons of water and is getting filled at a rate of 3 gallons per minute. How many gallons of water will be in the tank after 4 minutes?

Match each representation (a table or an expression) with situation 1 or situation 2. Be prepared to explain how the table or expression answers the question.

a. $8 \cdot 3 \cdot 3 \cdot 3 \cdot 3$

b.

x	0	1	2	3	4
y	80	240	720	2,160	6,480

c. $80 + 3 + 3 + 3 + 3$

d. $80 + 4 \cdot 3$

e.

x	0	1	2	3	4
y	80	83	86	89	92

f. $80 \cdot 81$

g. $80 + 12$

h. $80 \cdot 3^4$

Lesson Debrief

Lesson 4 Summary and Glossary

Here are two tables representing two different situations.

- A student runs errands for a neighbor every week. The table shows the pay he may receive, in dollars, in any given week.

Number of errands	Pay in dollars	Difference from previous pay
0	10	
1	15	$15-10=5$
2	20	$20-15=5$
3	25	$25-20=5$
4	30	$30-25=5$

- A student at a high school heard a rumor that a celebrity will be speaking at graduation. The table shows how the rumor is spreading over time, in days.

Day	People who have heard the rumor	Factor from previous number of people
0	1	
1	5	$5\div 1=5$
2	25	$25\div 5=5$
3	125	$125\div 25=5$
4	625	$625\div 125=5$

Once we recognize how these patterns change, we can describe them mathematically. This allows us to understand their behavior, extend the patterns, and make predictions.

In upcoming lessons, we will continue to describe and represent these patterns and use them to solve problems.

Unit 6 Lesson 4 Practice Problems

1. A population of ants was 10,000 at the start of April. Since then, it has tripled each month.
   a. Complete the table.
   b. What do you notice about the population differences from month to month?
   c. If there are n ants one month, how many ants will there be a month later?

Months since April	Number of ants
0	
1	
2	
3	
4	

2. A swimming pool contains 500 gallons of water. A hose is turned on, and it fills the pool at a rate of 24 gallons per minute. Which expression represents the amount of water in the pool, in gallons, after 8 minutes?
   a. $500 \cdot 24 \cdot 8$
   b. $500 + 24 + 8$
   c. $500 + 24 \cdot 8$
   d. $500 \cdot 24^8$

3. The population of a city is 100,000. It doubles each decade for 5 decades. Select **all** expressions that represent the population of the city after 5 decades.
   a. $32,000$
   b. $320,000$
   c. $100,000 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$
   d. $100,000 \cdot 5^2$
   e. $100,000 \cdot 2^5$

4. The table shows the height, in centimeters, of the water in a swimming pool at different times since the pool started to be filled.

   a. Does the height of the water increase by the same amount each minute? Explain how you know.

Minutes	Height (centimeters)
0	150
1	150.5
2	151
3	151.5

   b. Does the height of the water increase by the same factor each minute? Explain how you know.

5. Bank account C starts with \$10 and doubles each week. Bank account D starts with \$1,000 and grows by \$500 each week. When will account C contain more money than account D? Explain your reasoning.

(From Unit 6, Lesson 3)

6. Rewrite each expression using the fewest number of exponents. Show or explain your reasoning.

   a. $(x^6y^5)^2(2x3y^2)$

   b. $\frac{12m^4n^2}{3m^6n}$

   c. $(5x^{-3})(5x^0)(5x)$

(From Unit 6, Lessons 1 and 2)

7. Suppose C is a rule that takes time as the input and gives your class on Monday as the output. For example, $C(10:15) =$ Biology.

   a. Write three sample input-output pairs for C.

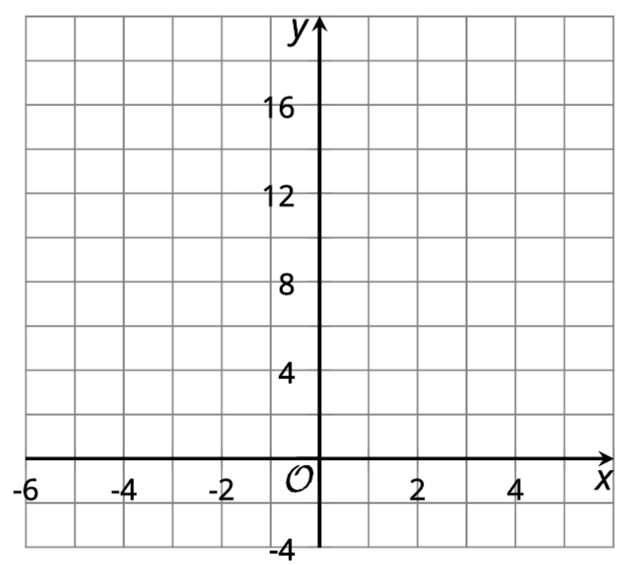
   b. Does each input to C have exactly one output? Explain how you know.

   c. Explain why C is a function.

(From Unit 5)

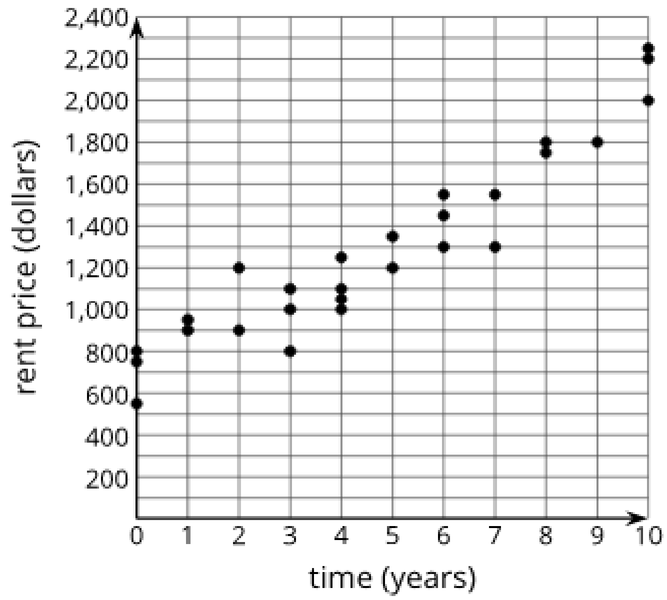
8. The rule that defines function f is $f(x) = x^2 + 1$. Complete the table. Then, sketch a graph of function f.

x	$f(x)$
-4	17
-2	
0	
2	
4	



(From Unit 5)

9. The scatter plot shows the rent prices for apartments in a large city over ten years.



   a. The regression equation is $y = 134.02x + 655.40$, where y represents the rent price in dollars, and x the time in years. Use it to estimate the rent price after 8 years. Show your reasoning.

   b. Use the best fit line to estimate the number of years it will take the rent price to equal $2,500. Show your reasoning.

(From Unit 4)

10. One horse can run $\frac{3}{4}$ of a mile in 1.5 minutes. Another runs $2\frac{1}{4}$ miles in 4.5 minutes. Assuming the horses run at a constant speed, how long does it take a horse to run 1 mile?

(Addressing NC.7.RP.3)

Lesson 5: Representing Exponential Growth

Learning Targets

- I can explain the connections between an equation and a graph that represents exponential growth.
- I can write and interpret an equation that represents exponential growth.

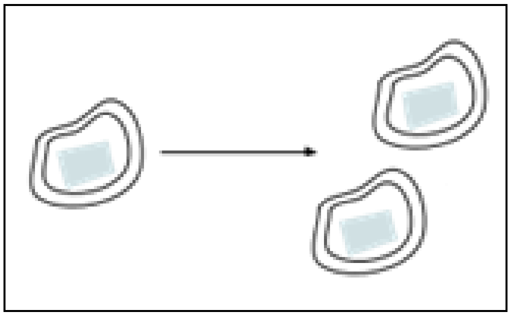
Bridge

Evaluate each expression.[1]

1. $7 + 2^3$
2. $9 \cdot 3^1$
3. $20 - 2^4$
4. $2 \cdot 6^2$
5. $8 \cdot (\frac{1}{2})^2$
6. $\frac{1}{3} \cdot 3^3$
7. $(\frac{1}{5} \cdot 5)^5$

Warm-up: Splitting Bacteria

There are some bacteria in a dish. Every hour, each bacterium splits into two bacteria.



1. This diagram shows the number of bacteria in hour 0 and then in hour 1. Draw what happens in hours 2 and 3.
2. How many bacteria are there in hours 2 and 3?

[1] Adapted from IM 6–8 Math https://curriculum.illustrativemathematics.org/MS/index.html, which was originally developed by Open Up Resources and authored by Illustrative Mathematics, and is copyright 2017–2019 by Open Up Resources. It is licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0). OUR's 6–8 Math Curriculum is available at https://openupresources.org/math-curriculum/. Adaptations and updates to IM 6–8 Math are copyright 2019 by Illustrative Mathematics, and are licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0).

Activity 1: HeLa Cells

1. In a medical research lab, 500 HeLa cells double approximately every 24 hours (1 day).

Day	Expression to determine the number of HeLa cells	Number of HeLa cells (evaluate your expression in the middle column)
0	500	
1	$500 \cdot$	
2	$500 \cdot$	
3	$500 \cdot$	
6	$500 \cdot$	
t	$500 \cdot$	

   a. In the middle column, the expression to show how to find the number of HeLa cells after each day has been started for you. Complete each expression in the middle column.

   b. Evaluate your expression in the third column.

   c. Write an equation relating n, the number of cells, to t, the number of days.

   d. Use your equation to find n when t is 0. What does this value of n mean in this situation?

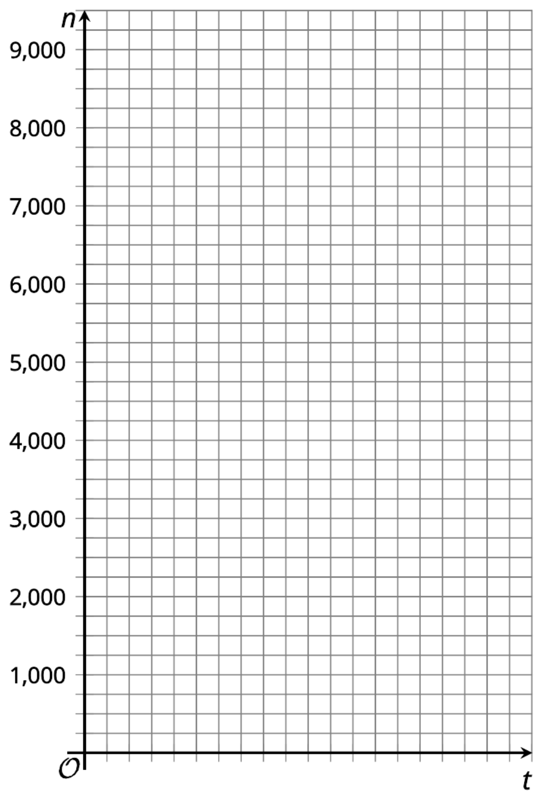
2. In a different medical research lab, a population of single-cell parasites also reproduces. An equation which gives the number of parasites, p, after t days is $p = 100 \cdot 3^t$. Explain what the numbers 100 and 3 mean in this situation.

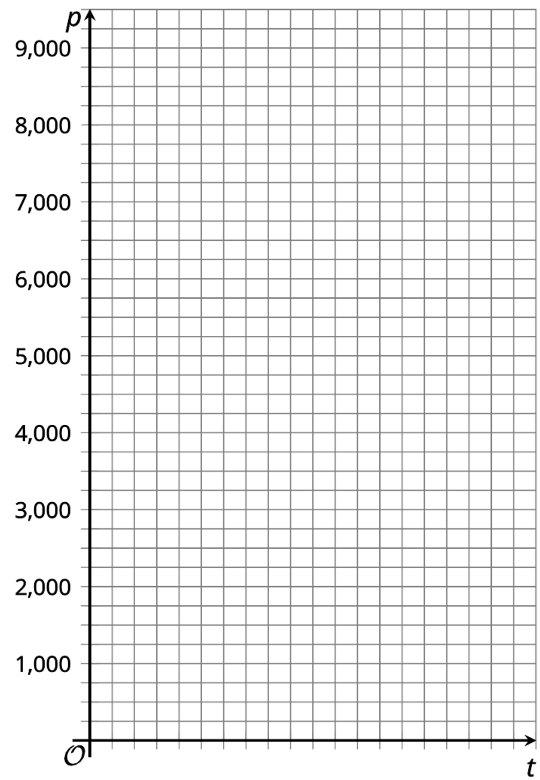
Activity 2: Graphing the HeLa Cells

1. Refer back to your work in the table of the previous task. Use that information and the given coordinate planes to graph the following:

    a. Graph (t, n) when t is 0, 1, 2, 3, and 4.

    b. Graph (t, p) when t is 0, 1, 2, 3, and 4. (If you get stuck, you can create a table.)





2. On the graph of n, where can you see each number that appears in the equation?

3. On the graph of p, where can you see each number that appears in the equation?

Lesson Debrief

Lesson 5 Summary and Glossary

A situation where a quantity increases through repeated multiplication by the same amount is called **exponential growth**. This multiplier is called the **growth factor**.

Exponential growth: The tendency of something to increase exponentially; that is, the quantity over time can be described by repeated multiplication by a number greater than 1.

Growth factor: Quantities changing exponentially can be described as repeated multiplication by a factor, or, in symbols, by the expression $a \cdot b^x$. The multiplier b is called the growth factor when it is greater than 1.

Suppose a population of cells starts at 500 and triples every day. The number of cells each day can be calculated as follows:

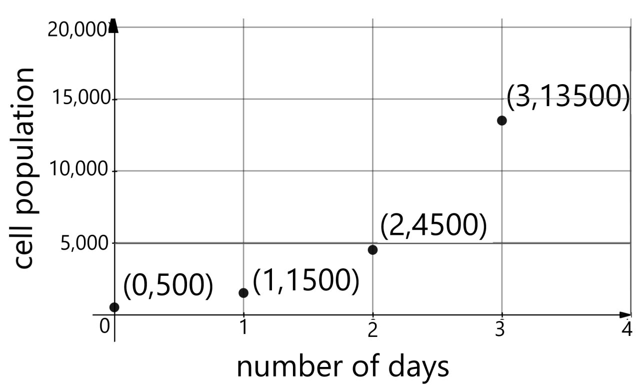
Number of days	Number of cells
0	500
1	$1,500$ (or $500 \cdot 3$)
2	$4,500$ (or $500 \cdot 3 \cdot 3$, or $500 \cdot 3^2$)
3	$13,500$ (or $500 \cdot 3 \cdot 3 \cdot 3$, or $500 \cdot 3^3$)
d	$500 \cdot 3^d$

We can see that the number of cells (p) is changing exponentially, and that p can be found by multiplying 500 by 3 as many times as the number of days (d) since the 500 cells were observed. The growth factor is 3. To model this situation, we can write this equation: $p = 500 \cdot 3^d$.

The equation can be used to find the population on any day: including day 0, when the population was first measured. On day 0, the population is $500 \cdot 3^0$. Since $3^0 = 1$, this is $500 \cdot 1$ or 500.

Here is a graph of the daily cell population. The point $(0, 500)$ on the graph means that on day 0, the population is 500.

Each point marked on the graph is 3 times higher on the graph than the previous point. $(1, 1500)$ is 3 times higher than $(0, 500)$, and $(2, 4500)$ is 3 times higher than $(1, 1500)$.

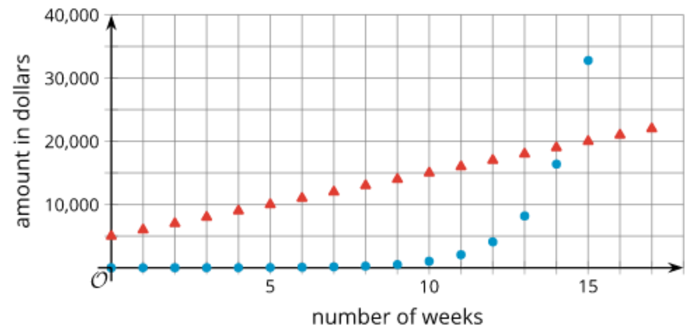


Unit 6 Lesson 5 Practice Problems

1. Bank account A starts with \$5,000 and grows by \$1,000 each week. Bank account B starts with \$1 and doubles each week.

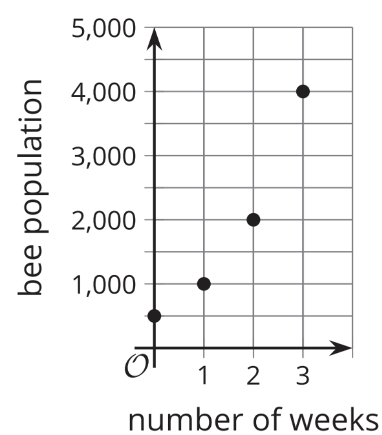
    a. Which account has more money after one week? After two weeks?

    b. Here is a graph showing the two account balances. Which graph corresponds to which situation? Explain how you know.



    c. Given a choice, which of the two accounts would you choose? Explain your reasoning.

2. A bee population is measured each week, and the results are plotted on the graph.



    a. What is the bee population when it is first measured?

    b. Is the bee population growing by the same factor each week? Explain how you know.

    c. What is an equation that models the bee population, b, w weeks after it is first measured?

3. A bond is initially bought for \$250. It doubles in value every decade.
   a. Complete the table.
   b. How many decades does it take before the bond is worth more than \$10,000?
   c. Write an equation relating v, the value of the bond, to d, the number of decades since the bond was bought.

Decades since bond is bought	Dollar value of bond
0	
1	
2	
3	
d	

4. A sea turtle population p is modeled by the equation $p = 400 \cdot 2^y$ where y is the number of years since the population was first measured.
   a. How many turtles are in the population when it is first measured? Where do you see this in the equation?
   b. Is the population increasing or decreasing? How can you tell from the equation?
   c. When will the turtle population reach 4000? Explain how you know.

5. Which expression is equal to $4^0 \cdot 4^2$?
    a. 0
    b. 1
    c. 16
    d. 64

(From Unit 6, Lesson 2)

6. Select **all** expressions equivalent to 3^8.
    a. 8^3
    b. $\frac{3^{10}}{3^2}$
    c. $3 \cdot 8$
    d. $(3^4)^2$
    e. $(3 \cdot 3)^4$
    f. $\frac{1}{3^{-8}}$

(From Unit 6, Lesson 2)

7. Without using a calculator, take a guess: Which of the three expressions is the largest: A, B, or C?

A	B	C
$9^5 \cdot 9^3$	$(9^5)^3$	$9^5 + 9^3$

Now, check your guess using a calculator. Why is the largest expression the largest, based on the exponents?

(From Unit 6, Lesson 1)

8. Function F is defined so that its output $F(t)$ is the number of followers on a social media account t days after setup of the account.

   a. Explain the meaning of $F(30) = 8,950$ in this situation.

   b. Explain the meaning of $F(0) = 0$.

   c. Write a statement about function F that represents the fact that there were 28,800 followers 110 days after the set up of the account.

   d. Explain the meaning of t in the equation $F(t) = 100,000$.

(From Unit 5)

9. Match each equation in the first list to an equation in the second list that has the same solution.

   a. $y = \frac{2}{5}x + 2$
   b. $x = -5 - 2.5y$
   c. $y = \frac{10}{5} - 0.4x$
   d. $2x = 10 - 5y$
   e. $-5y = 2x + 10$
   f. $x = 5 - \frac{5}{2}y$

   1. $2x + 5y = 10$
   2. $-2x - 5y = 10$
   3. $-2x + 5y = 10$

(From Unit 2)

10. Evaluate each expression if $x = 3$.

a. $2^x \, 2^x$

b. $x^2 \, x^2$

c. $1^x \, 1^x$

d. $x^1 \, x^1$

e. $(\frac{1}{2})^x \, (\frac{1}{2})^x$

(Addressing NC.6.EE.2)[2]

11. Evaluate each expression for the given value of each variable.

a. $2 + x^3$, x is $3 \, 3$

b. x^2, x is $\frac{1}{2} \, \frac{1}{2}$

c. $3x^2 + y$, x is 5 and y is $3 \, 3$

d. $10y + x^2$, x is 6 and y is $4 \, 4$

(Addressing NC.6.EE.2)[3]

[2] Adapted from IM 6–8 Math https://curriculum.illustrativemathematics.org/MS/index.html, which was originally developed by Open Up Resources and authored by Illustrative Mathematics, and is copyright 2017–2019 by Open Up Resources. It is licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0). OUR's 6–8 Math Curriculum is available at https://openupresources.org/math-curriculum/. Adaptations and updates to IM 6–8 Math are copyright 2019 by Illustrative Mathematics, and are licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0).
[3] Adapted from IM 6–8 Math https://curriculum.illustrativemathematics.org/MS/index.html (see above).

Lesson 6: Understanding Decay

Learning Targets

- I know the meanings of “exponential growth” and “exponential decay.”
- I can use only multiplication to represent "decreasing a quantity by a fraction of itself."
- I can write an expression or equation to represent a quantity that decays exponentially.

Bridge

Multiply:

1. $2 \cdot 1$
2. $2 \cdot \frac{1}{2}$
3. $2 \cdot \frac{3}{2}$
4. $\frac{5}{8} \cdot 1$
5. $\frac{5}{8} \cdot \frac{1}{2}$
6. $\frac{5}{8} \cdot \frac{3}{2}$
7. What do you notice or wonder about the results when you multiply by $\frac{1}{2}$ compared to when you multiply by $\frac{3}{2}$?

Warm-up: Two Tables

What do you notice? What do you wonder?

Table A

x	y
0	2
1	3
2	$\frac{9}{2}$
3	$\frac{27}{4}$
4	$\frac{81}{8}$

Table B

x	y
0	1600
1	800
2	400
3	200
4	100

Activity 1: What's Left?

1. Here is one way to think about how much Diego has left after spending $\frac{1}{4}$ of $\$100$. Explain each step.
   - Step 1: $100 - \frac{1}{4} \cdot 100$
   - Step 2: $100(1 - \frac{1}{4})$
   - Step 3: $100 \cdot \frac{3}{4}$
   - Step 4: $\frac{3}{4} \cdot 100$

2. A person makes $\$1,800$ per month, but $\frac{1}{3}$ of that amount goes to her rent. What two numbers can you multiply to find out how much she has after paying her rent?

3. Write an expression that only uses multiplication and that is equivalent to "x reduced by $\frac{1}{8}$ of x."

Activity 2: Value of a Vehicle

Every year after a new car is purchased, it loses $\frac{1}{3}$ of its value. Let's say that a new car costs $\$18,000$.

1. A buyer worries that the car will be worth nothing in three years due to *depreciation*. Do you agree? Explain your reasoning.

2. Write an expression to show how to find the value of the car for each year listed in the table.

Year	Expression to calculate the value of car (dollars)	Value of car (dollars)
0	18,000	18,000
1		
2		
3		
6		
t		

3. Write an equation relating the value of the car in dollars, v, to the number of years, t.

4. Use your equation to find v when t is 0. What does this value of v mean in this situation?

5. A different car loses value at a different rate. The value of this different car in dollars, d, after t years can be represented by the equation $d = 10,000 \cdot (\frac{4}{5})^t$.

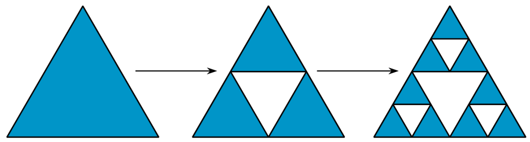
   a. Explain what the numbers $10,000$ and $\frac{4}{5}$ mean in this situation.

   b. Write an expression to show how to find the value of the car for each year listed in the table.

Year	Expression to calculate the value of car (dollars)	Value of car (dollars)
0	10,000	10,000
1		
2		
3		
4		
5		

Are You Ready For More?

Start with an equilateral triangle with area 1 square unit, divide it into four congruent pieces as in the figure, and remove the middle one. Then, repeat this process with each of the remaining pieces. Repeat this process over and over for the remaining pieces. The figure shows the first two steps of this construction.



What fraction of the area is removed each time? How much area is removed after the n-th step? Use a calculator to find out how much area remains in the triangle after 50 such steps have been taken.

Lesson Debrief

Lesson 6 Summary and Glossary

Sometimes a quantity grows by the same factor at regular intervals. For example, a population doubles every year. Sometimes a quantity *decreases* by the same factor at regular intervals. For example, a car might lose one third of its value every year. When an object loses value over time, we say that it **depreciates**.

> **Depreciate**: To lose value over time. For example, a car is worth less money the older it is.

Let's look at a situation where a quantity decreases by the same factor at regular intervals. Suppose a bacteria population starts at $100,000$ and $\frac{1}{4}$ of the population dies each day. The population one day later is $100,000 - \frac{1}{4} \cdot 100,000$, which can be written as $100,000(1 - \frac{1}{4})$. The population after one day is $\frac{3}{4}$ of $100,000$: or $75,000$. The population after two days is $\frac{3}{4} \cdot 75,000$. Here are some further values for the bacteria population

Number of days	Bacteria population
0	$100,000$
1	$75,000$ (or $100,000 \cdot \frac{3}{4}$)
2	$56,250$ (or $100,000 \cdot \frac{3}{4} \cdot \frac{3}{4}$, or $100,000 \cdot (\frac{3}{4})^2$)
3	about $42,188$ (or $100,000 \cdot \frac{3}{4} \cdot \frac{3}{4} \cdot \frac{3}{4}$, or $100,000 \cdot (\frac{3}{4})^3$)

In general, d days after the bacteria population was $100,000$, the population p is given by the equation: $p = 100,000 \cdot (\frac{3}{4})^d$, with one factor of $\frac{3}{4}$ for each day.

Situations with quantities that decrease exponentially are described as **exponential decay**. The multiplier ($\frac{3}{4}$ in this case) is called the **decay factor**.

> **Exponential decay:** The tendency of a quantity to decrease exponentially: that is, the quantity remaining over time can be described by repeated multiplication by a number between 0 and 1.
>
> **Decay factor:** Quantities changing exponentially can be described as repeated multiplication by a factor, or, in symbols, by the expression $a * b^x$. The multiplier *b* is referred to as the decay factor when this number is between 0 and 1.

Unit 6 Lesson 6 Practice Problems

1. A new bicycle sells for $\$300$. It is on sale for $\frac{1}{4}$ off the regular price. Select **all** the expressions that represent the sale price of the bicycle in dollars.

   a. $300 \cdot \frac{1}{4}$

   b. $300 \cdot \frac{3}{4}$

   c. $300 \cdot (1 - \frac{1}{4})$

   d. $300 - \frac{1}{4}$

   e. $300 - \frac{1}{4} \cdot 300$

2. A computer costs $\$800$. It loses $\frac{1}{4}$ of its value every year after it is purchased.

   a. Complete the table to show the value of the computer at the listed times.

Time (years)	Value of computer (dollars)
0	
1	
2	
3	
t	

   b. Write an equation representing the value, v, of the computer, t years after it is purchased.

   c. Use your equation to find v when t is 5. What does this value of v mean?

3. A piece of paper is folded into thirds multiple times. The area, A, of the piece of paper in square inches, after n folds, is $A = 90 \cdot (\frac{1}{3})^n$.

   a. What is the value of A when $n = 0$? What does this mean in this situation?

   b. How many folds are needed before the area is less than 1 square inch?

   c. The area of another piece of paper in square inches, after n folds, is given by $B = 100 \cdot (\frac{1}{2})^n$. What do the numbers 100 and $\frac{1}{2}$ mean in this situation?

4. At the beginning of April, a colony of ants has a population of 5,000.

   a. The colony decreases by $\frac{1}{5}$ during April. Write an expression for the ant population at the end of April.

   b. During May, the colony decreases again by $\frac{1}{5}$ of its size. Write an expression for the ant population at the end of May.

   c. The colony continues to decrease by $\frac{1}{5}$ of its size each month. Write an expression for the ant population after 6 months.

5. Lin starts with 13 mystery novels. Each month, she gets two more. Select **all** expressions that represent the total number of Lin's mystery novels after 3 months.

   a. $13+2+2+2$

   b. $13\cdot 2\cdot 2\cdot 2$

   c. $13\cdot 8$

   d. $13+6$

   e. 19

6. An odometer is the part of a car's dashboard that shows the number of miles a car has traveled in its lifetime. Before a road trip, a car's odometer reads 15,000 miles. During the trip, the car travels 65 miles per hour.

   a. Complete the table.

   b. What do you notice about the differences of the odometer readings each hour?

   c. If the odometer reads n miles at a particular hour, what will it read one hour later?

Duration of trip (hours)	Odometer reading (miles)
0	
1	
2	
3	
4	
5	

(From Unit 6, Lesson 4)

7. Rewrite and simplify the following expressions so all exponents are positive:

   a. $\frac{5x^4y^{-2}z}{15x^{-3}yz^4}$

   b. $\left(\frac{2}{3}a^4b^2c^{-4}\right)^2$

(From Unit 6, Lesson 2)

8. A function multiplies its input by $\frac{3}{4}$ then adds 7 to get its output. Use function notation to represent this function.

(From Unit 5)

9. A function is defined by the equation $f(x) = 2x - 5$.

   a. What is $f(0)$?

   b. What is $f(\frac{1}{2})$?

   c. What is $f(100)$?

   d. What is x when $f(x) = 9$?

(From Unit 5)

10. A group of students is collecting 16 oz and 28 oz jars of peanut butter to donate to a food bank. At the end of the collection period, they donated 1,876 oz of peanut butter and a total of 82 jars of peanut butter to the food bank.

    a. Write a system of equations that represents the constraints in this situation. Be sure to specify the variables that you use.

    b. How many 16 oz jars and how many 28 oz jars of peanut butter were donated to the food bank? Explain or show how you know.

(From Unit 2)

Lesson 7: Representing Exponential Decay

Learning Targets

- I can find a decay factor from a graph and write an equation to represent exponential decay.
- I can graph equations that represent quantities that change by a factor between 0 and 1.
- I can explain the meanings of a and b in an equation that represents exponential decay and is written as $y = a \cdot b^x$.

Bridge

Write each expression using an exponent.[1]

1. $1 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7$
2. $1 \cdot \left(\frac{4}{5}\right) \cdot \left(\frac{4}{5}\right) \cdot \left(\frac{4}{5}\right) \cdot \left(\frac{4}{5}\right) \cdot \left(\frac{4}{5}\right)$
3. $1 \cdot (9.3) \cdot (9.3) \cdot (9.3) \cdot (9.3) \cdot (9.3) \cdot (9.3) \cdot (9.3) \cdot (9.3)$
4. The number of coins Jada will have on the eighth day if she starts with one coin and the number of coins doubles every day. (She has two coins on the first day of the doubling.)

Warm-up: Two Other Tables

Use the patterns you notice to complete the tables. Show your reasoning.

Table A

x	0	1	2	3	4	25
y	2.5	10	17.5	25		

Table B

x	0	1	2	3	4	25
y	2.5	10	40	160		

[1] Adapted from IM 6–8 Math https://curriculum.illustrativemathematics.org/MS/index.html, which was originally developed by Open Up Resources and authored by Illustrative Mathematics, and is copyright 2017–2019 by Open Up Resources. It is licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0). OUR's 6–8 Math Curriculum is available at https://openupresources.org/math-curriculum/. Adaptations and updates to IM 6–8 Math are copyright 2019 by Illustrative Mathematics, and are licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0).

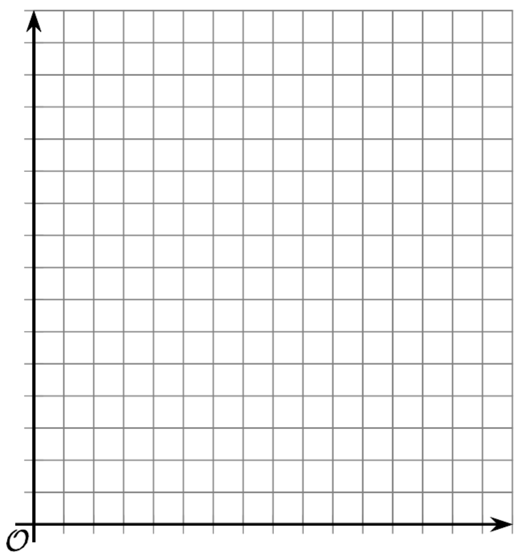
Activity 1: The Algae Bloom

In order to control an algae bloom in a lake, scientists introduce some treatment products.

Once the treatment begins, the area covered by algae A, in square yards, is given by the equation $A = 240 \cdot (\frac{1}{3})^t$. Time, t, is measured in weeks.

1. In the equation, what does the 240 tell us about the algae? What does the $\frac{1}{3}$ tell us?

2. Create a graph to represent $A = 240 \cdot (\frac{1}{3})^t$ when t is 0, 1, 2, 3, and 4. Think carefully about how you choose the scale for the axes. If you get stuck, consider creating a table of values.



t	A

3. Approximately how many square yards will the algae cover after 2.5 weeks? Explain your reasoning.

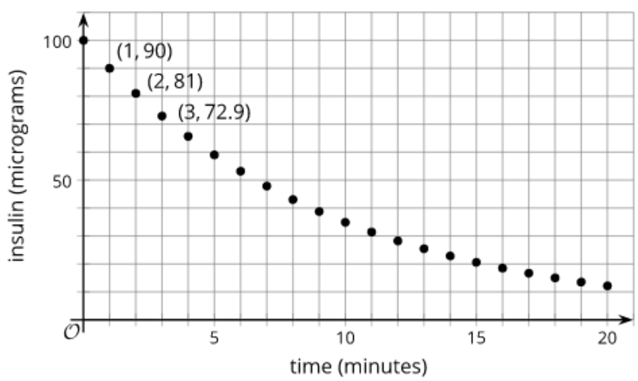
4. Use the rule to evaluate A when t is -1. Do the values make sense in this situation? Explain.

Are You Ready For More?

The scientists estimate that to keep the algae bloom from spreading after the treatment concludes, they will need to reduce the area covered to under one square foot. How many weeks should they run the treatment in order to achieve this?

Activity 2: Insulin in the Body

A patient who is diabetic receives 100 micrograms of insulin. The graph shows the amount of insulin, in micrograms, remaining in his bloodstream over time, in minutes.



1. Scientists have found that the amount of insulin in a patient's body changes exponentially. How can you check if the graph supports the scientists' claim?

2. How much insulin broke down in the first minute? What fraction of the original insulin is that?

3. How much insulin broke down in the second minute? What fraction is that of the amount one minute earlier?

4. What fraction of insulin remains in the bloodstream for each minute that passes? Explain your reasoning.

5. Complete the table to show the predicted amount of insulin 4 and 5 minutes after injection.

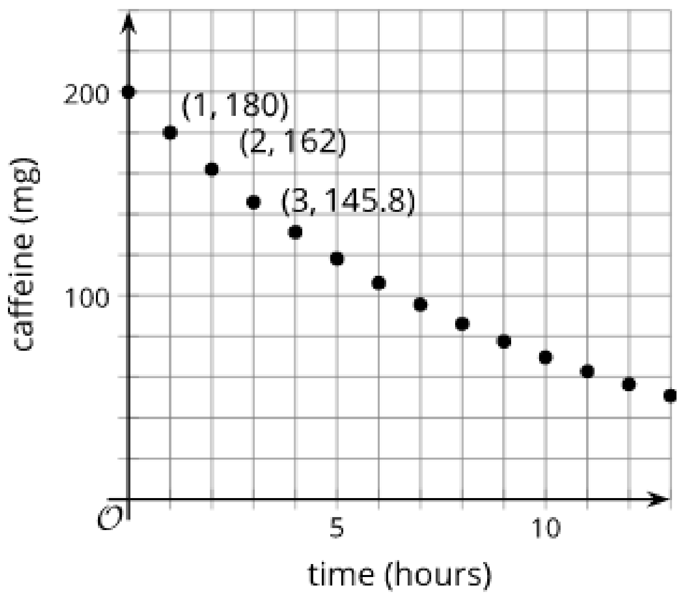
Time after injection (minutes)	0	1	2	3	4	5
Insulin in the bloodstream (micrograms)	100	90	81	72.9		

6. Describe how you would find how many micrograms of insulin remain in his bloodstream after 10 minutes. After m minutes?

Lesson Debrief

Lesson 7 Summary and Glossary

Here is a graph showing the amount of caffeine in a person's body, measured in milligrams, over a period of time, measured in hours. We are told that the amount of caffeine in the person's body changes exponentially.



The graph includes the point $(0, 200)$. This means that there were 200 milligrams of caffeine in the person's body when it was initially measured. The point $(1, 180)$ tells us there were 180 milligrams of caffeine 1 hour later. Between 6 and 7 hours after the initial measurement, the amount of caffeine in the body fell below 100 milligrams.

We can use the graph to find out what fraction of caffeine remains in the body each hour. Notice that $\frac{180}{200} = \frac{9}{10}$ and $\frac{162}{180} = \frac{9}{10}$. As each hour passes, the amount of caffeine that stays in the body is multiplied by a factor of $\frac{9}{10}$.

If y is the amount of caffeine, in milligrams, and t is time, in hours, then this situation is modeled by the equation: $y = 200 \cdot \left(\frac{9}{10}\right)^t$.

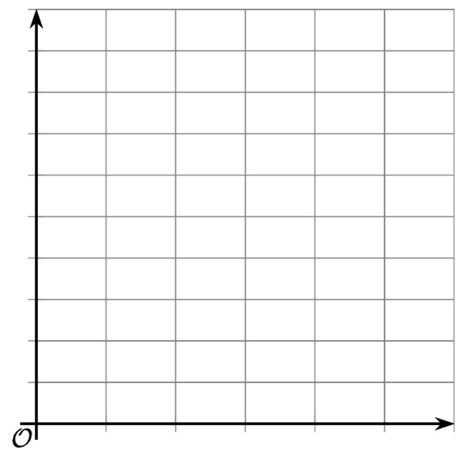
Unit 6 Lesson 7 Practice Problems

1. A population p of migrating butterflies satisfies the equation $p = 100,000 \cdot (\frac{4}{5})^w$ where w is the number of weeks since they began their migration.

   a. Complete the table with the population after different numbers of weeks.

w	0	1	2	3	4
p					

   b. Graph the butterfly population. Think carefully about how to choose a scale for the axes.

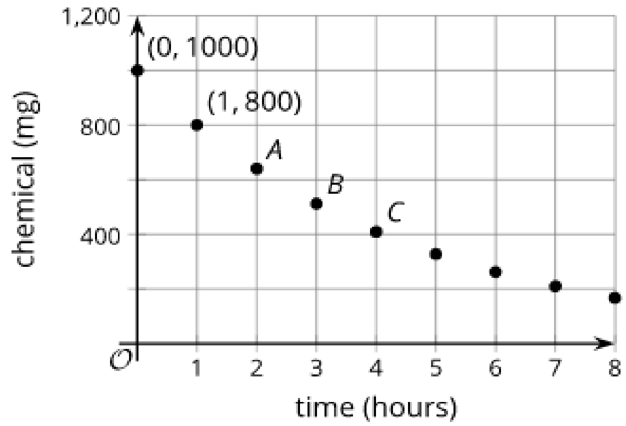


   c. What is the vertical intercept of the graph? What does it tell you about the butterfly population?

   d. About when does the butterfly population reach 50,000?

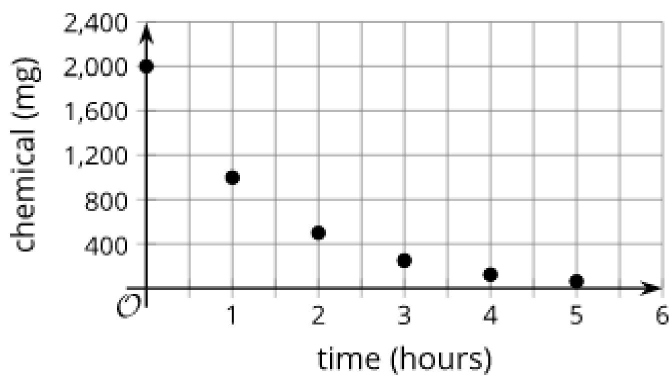
2. The graph shows the amount of a chemical in a water sample. It is decreasing exponentially.

   Find the coordinates of the points labeled A, B, and C. Explain your reasoning.



3. The graph shows the amount of a chemical in a patient's body at different times measured in hours since the levels were first checked.

   Could the amount of this chemical in the patient be decaying exponentially? Explain how you know.



4. The height of a plant is 7 mm. It doubles each week. Select **all** expressions that represent the height of the plant, in mm, after 4 weeks.

   a. $7+4\cdot2$

   b. $7\cdot2^4$

   c. $2+7^4$

   d. $7\cdot2\cdot2\cdot2\cdot2$

   e. $7\cdot2\cdot4$

5. At the beginning of January, 300 people have read a new book. The number of people who have read the book doubles each month.

   a. Use this information to complete the table.

   b. What do you notice about the difference in the number of people who have read the book from month to month?

   c. What do you notice about the factor by which the number of people changes each month?

Number of months since the beginning of January	Number of people who have read the book
0	
1	
2	
3	
4	

   d. If n people have read the book one month, how many people will have read the book the following month?

6. The number of Netflix subscribers in Latin America has increased a lot in recent years.[2] The number of paid subscribers from 2018–2020 was:

Year	2018	2019	2020
Number of Subscribers (in millions)	26.08	31.42	37.54

Noah and Elena agree that the number of subscribers is probably increasing exponentially—everyone loves Netflix! Noah says the growth factor is about 20%, or 0.2, but Elena disagrees and says the growth factor is about 1.2. Who do you agree with? Explain your reasoning.

(From Unit 6, Lesson 5)

[2] Dean, B. (2021, February 25). *Netflix subscriber and growth statistics: How many people watch Netflix in 2021?* Backlinko. https://backlinko.com/netflix-users#netflix-payed-suscriber-growth

7. Researchers at the University of North Carolina are studying the spread of diseases. For a new bacterial disease, they are able to isolate one cell, and they watch as it divides into three cells over the first hour. The number of cells grows exponentially until there are 3^5 cells after 5 hours. The researchers are nervous about the constant growth, and they keep watching the bacteria grow. Over the next 4 hours, the number of cells continues to grow, multiplying the previous total by 3^4.

   Express the total number of cells after 9 hours first as an exponential expression and then as a whole number.

   (From Unit 6, Lesson 1)

8. Tyler creates a scatter plot that displays the relationship between the grams of food a hamster eats, x, and the total number of rotations that the hamster's wheel makes, y. They create a line of best fit and find that the residual for the point $(1.2, 1364)$ is 117. Interpret the meaning of 117 in the context of the problem.

   (From Unit 4)

9. Solve each system of equations.

a. $\begin{cases} x + y = 2 \\ -3x - y = 5 \end{cases}$

b. $\begin{cases} \frac{1}{2}x + 2y = -13 \\ x - 4y = 8 \end{cases}$

(From Unit 3)

10. $8^{12} \cdot 8^{-20} = 8^{x}$

What value of x makes the equation true?

(Addressing NC.8.EE.1)

Lesson 8: Analyzing Graphs

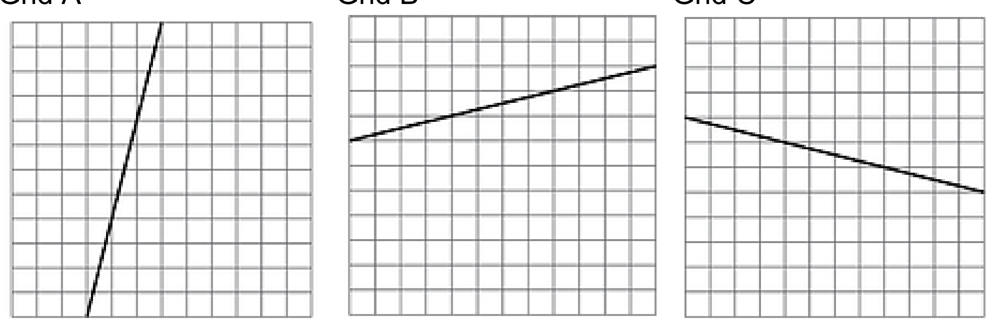
Learning Targets

- I can tell whether a situation involves exponential growth or exponential decay based on a description or a graph.
- I can use graphs to compare and contrast situations that involve exponential decay.
- I can use information from a graph to write an equation that represents exponential decay.

Bridge

1. Each square on a grid represents 1 unit on each side. Match the numbers with the slopes of the lines.[1]

Grid A Grid B Grid C



1. $-\frac{1}{4}$
2. $\frac{1}{4}$
3. 4

2. Explain how someone could determine the match to grid C just by looking at the graphs and the answer choices.

Warm-up: Fractions and Decimals

A hospital offers two different medications to treat pain: medication A and medication B.

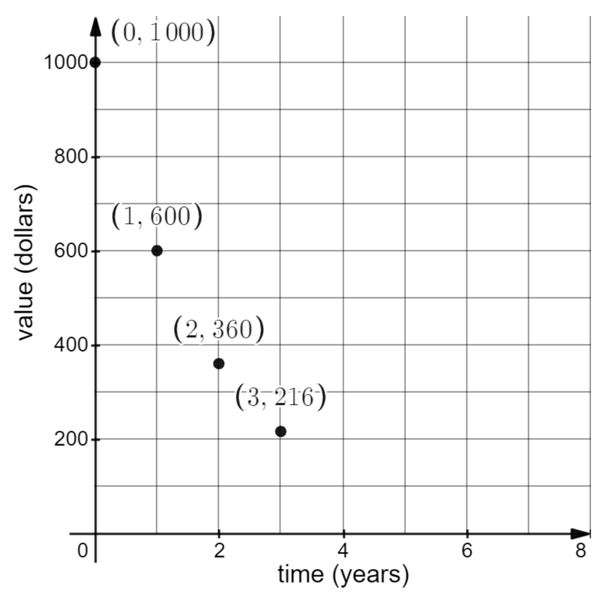
- The amount of medication A that remains in the body each hour is $\frac{5}{7}$ of the amount in the previous hour.
- The amount of medication B that remains in the body each hour is $\frac{2}{5}$ of the amount in the previous hour.

Which medication leaves the body the fastest? Explain your reasoning.

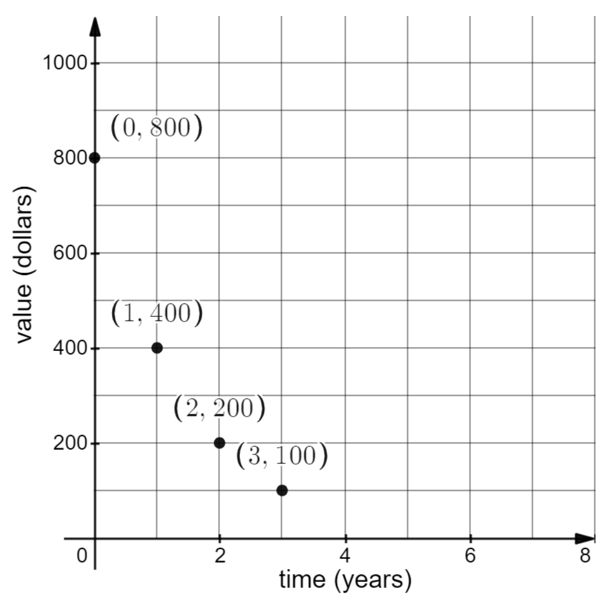
Activity 1: Falling and Falling

The value of some cell phones changes exponentially after initial release. Here are graphs showing the depreciation of two phones 1, 2, and 3 years after they were released.

Phone A

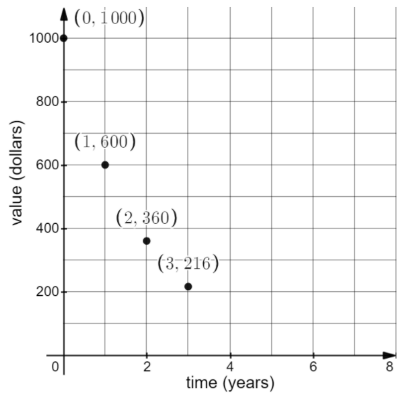


Phone B

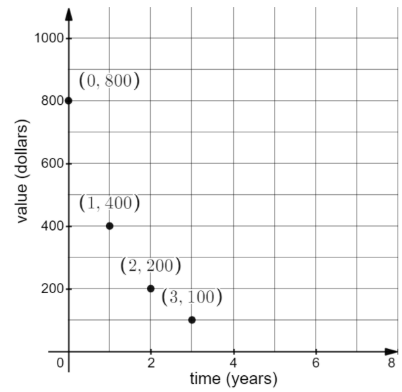


1. Which phone is more expensive to buy when it is first released?

Phone A



Phone B



2. How does the value of each phone change with every passing year?

3. Which one is decreasing in value more quickly? Explain or show how you know.

4. If the phones continue to depreciate by the same factor each year, what will the value of each phone be 4 years after its initial release?

5. For each cell phone, write an equation that relates the value of the phone in dollars to the years since release, t. Use v for the value of phone A and w for the value of phone B.

Are You Ready For More?

When given data, it is not always clear how to best model it. In this case, we were told the value of the cell phones was changing exponentially. Suppose, however, we were instead given only the initial values of the cell phones when released and the values after each of the first three years.

1. Use technology to compute the best fit line for each cell phone. Round any numbers to the nearest dollar.

2. Explain why, in this situation, an exponential model might be more appropriate than the linear model you just created.

Activity 2: Matching Descriptions to Graphs

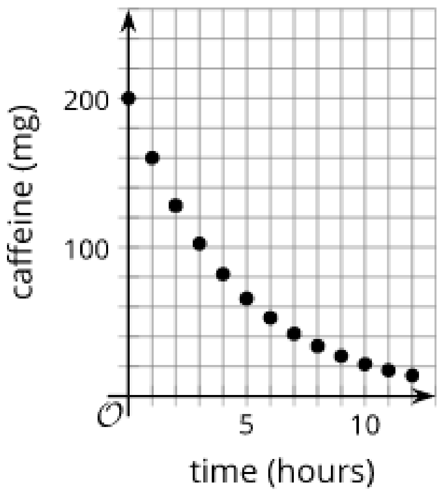
Your teacher will give you a set of cards containing descriptions of situations and graphs. Match each situation with a graph that represents it. Record your matches and be prepared to explain your reasoning.

Lesson Debrief

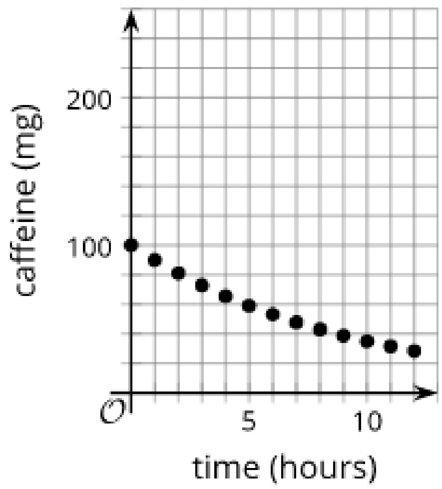
Lesson 8 Summary and Glossary

Graphs are useful for comparing relationships. Here are two graphs representing the amount of caffeine in the bloodstreams of Person A and Person B, in milligrams, at different times, measured hourly, after an initial measurement.

Person A



Person B

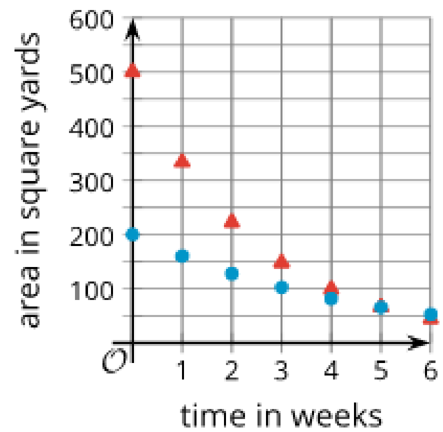


The graphs reveal interesting information about the amount of caffeine in each person over time:

- At the initial measurement, Person A’s bloodstream has more caffeine (200 milligrams) than Person B’s (100 milligrams).
- The caffeine in Person A's bloodstream decreases faster. It went from 200 to 160 milligrams in an hour. Because 160 is $\frac{8}{10}$ or $\frac{4}{5}$ of 200, the decay factor is $\frac{4}{5}$.
- The caffeine in Person B's bloodstream went from 100 to about 90 milligrams in an hour, so that decay factor is about $\frac{9}{10}$. This means that after each hour, a larger fraction of caffeine stays in Person B’s bloodstream than in Person A’s.
- Even though Person A started out with twice as much caffeine in their bloodstream, because of the decay factor, Person A’s bloodstream had less caffeine than Person B’s after 6 hours.

Unit 6 Lesson 8 Practice Problems

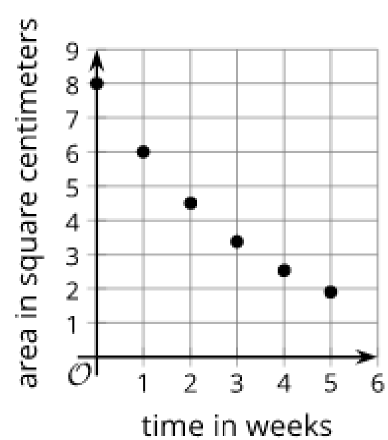
1. The two graphs show models characterized by exponential decay representing the area covered by two different algae blooms, in square yards, w weeks after different chemicals were applied.



   a. Which algae bloom covered a larger area when the chemicals were applied? Explain how you know.

   b. Which algae population is decreasing more rapidly? Explain how you know.

2. A medicine is applied to a burn on a patient's arm. The area of the burn in square centimeters decreases exponentially and is shown in the graph.



   a. What fraction of the burn area remains each week?

   b. Write an equation representing the area of the burn, a, after t weeks.

   c. What is the area of the burn after 7 weeks? Round to three decimal places.

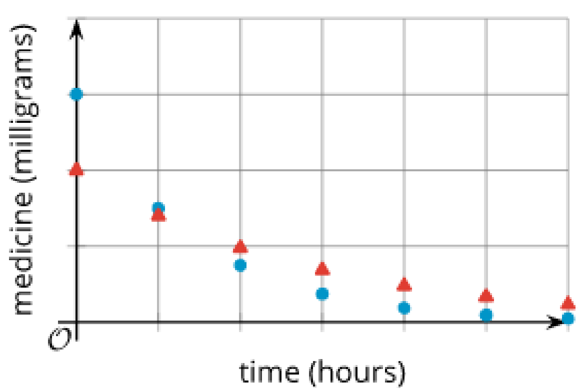
3.

a. The area of a sheet of paper is 100 square inches. Write an equation that gives the area, A, of the sheet of paper, in square inches, after being folded in half n times.

b. The area of another sheet of paper is 200 square inches. Write an equation that gives the area, B, of this sheet of paper, in square inches, after being folded into thirds n times.

c. Are the areas of the two sheets of paper ever the same after each being folded n times? Explain how you know.

4. The graphs show the amounts of medicine in two patients after receiving injections. The circles show the medicine in Patient A and the triangles show that in Patient B.



One equation that gives the amount of medicine in milligrams, m, in Patient A, h hours after an injection, is $m = 300(\frac{1}{2})^h$.

What could be an equation for the amount of medicine in Patient B?

a. $m = 500(\frac{3}{10})^h$

b. $m = 500(\frac{7}{10})^h$

c. $m = 200(\frac{3}{10})^h$

d. $m = 200(\frac{7}{10})^h$

5. Select **all** expressions that are equivalent to 3^8.

   a. $3^2 \cdot 3^4$

   b. $3^2 \cdot 3^6$

   c. $\frac{3^{16}}{3^2}$

   d. $\frac{3^{12}}{3^4}$

   e. $(3^4)^2$

   f. $(3^1)^7$

(From Unit 6, Lessons 1 and 2)

6. Priya simplifies $(\frac{x}{x^{-4}})^3$ to x^{15} using the following steps:

   - Step 1: $(\frac{x^3}{x^{-12}})$
   - Step 2: $(x^3 \cdot x^{12})$
   - Step 3: x^{15}

   Han simplifies the same expression to x^{15}, and he uses a different series of steps. What steps might Han have used?

(From Unit 6, Lesson 2)

7. (*Technology required.*) Use a graphing calculator to determine the equation of the line of best fit. Round numbers to two decimal places.

x	10	12	15	16	18	20	24
y	27	22	21	19	15	14	10

(From Unit 4)

8. The data set represents the number of students in several different classes who scored a perfect score on the most recent math test.

   12, 14, 15, 15, 17, 19, 20, 30

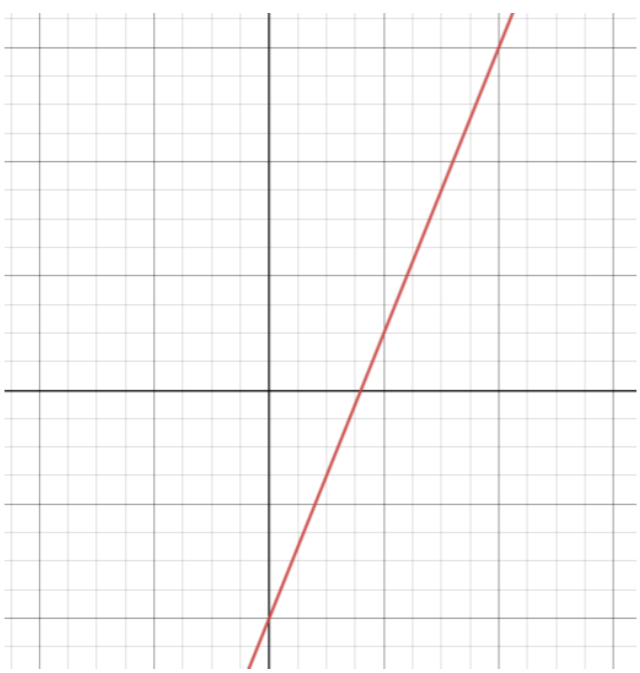
   a. What is the mean of the data set? Interpret this value in the situation.

   b. What is the median of the data set? Interpret this value in the situation.

   c. Is there an outlier? How does it impact the mean compared to the median?

(From Unit 1)

9. What is the slope of the line below? How do you know?



(Addressing NC.8.F.4)

Lesson 9 & 10: Checkpoint

Learning Targets

- I can continue to grow as a mathematician and challenge myself.
- I can share what I know mathematically.

Station C: Recalling Percent Change

1. You need to pay 8% tax on a car that costs $12,000. What will you end up paying in total? Show your reasoning.

2. Burritos are on sale for 30% off. Your favorite burrito normally costs $8.50. How much does it cost now? Show your reasoning.

3. A pair of shoes that originally costs $79 is on sale for 35% off. Does the expression $0.65(79)$ represent the sale price of the shoes (in dollars)? Explain your reasoning.

4. Come up with some strategies for mentally adding 15% to the total cost of an item.

5. Complete the table so that each row has a description and two different expressions that answer the question asked in the description. The second expression should use only multiplication. Be prepared to explain how the two expressions are equivalent.

Description and question	Expression 1	Expression 2 (using only multiplication)
A one-night stay at a hotel in Anaheim, CA costs \$160. Hotel room occupancy tax is 15%. What is the total cost of a one-night stay?	$160 + (0.15) \cdot 160$	
Teachers receive a 30% discount at a museum. An adult ticket costs \$24. How much would a teacher pay for admission into the museum?		$(0.7) \cdot 24$
The population of a city was 842,000 ten years ago. The city now has 2% more people than it had then. What is the population of the city now?		
After a major hurricane, 46% of the 90,500 households on an island lost their access to electricity. How many households still have electricity?		
	$754 - (0.21) \cdot 754$	
Two years ago, the number of students in a school was 150. Last year, the student population increased 8%. This year, it increased by 8% again. What is the number of students this year?		

Station D: The Legacy of Henrietta Lacks

In Unit 6, Lesson 5, Activity 1, you were introduced to HeLa cells, with a unique characteristic of doubling about every 24 hours, used for medical research. Take 10 minutes to learn more about Henrietta Lacks and HeLa cells, and then answer the following questions:

1. Suppose the family of Henrietta Lacks was given $1,000 as compensation for using HeLa cells for research once it was discovered how valuable they were in 1951. Use the question and prompts below to determine the amount this compensation would be worth to the family today if it accrued interest at a rate of 10% per year.
    a. If an amount of money grows 10% per year, what is the annual growth factor?
    b. Create an exponential equation for a function representing the value of compensation as a function of the number of years since 1951, with an initial value of $1,000, and the growth factor from part a.
    c. Calculate the value of $1,000 today, using the number of years since 1951.
2. Do you believe the family of Henrietta Lacks deserves compensation for the contributions she unknowingly made to modern medicine? If so, how much do you think is owed to the family? Share your reasoning.

3. Do you think requiring consent for research should be required, meaning an individual or family has the right to agree or refuse permission to use samples taken during treatment? Describe why you believe consent is a good or bad requirement for medical research.

Station E: I Know My Exponents!

1. In these puzzles, you will fill in the boxes to make true equations involving exponents. The example on the right shows a completed puzzle. Using only numbers 0–9, can you find two other combinations that work?

$$\boxed{3}^{\boxed{2}} = \boxed{9}$$

a. $\square^{\square} = \square$

b. $\square^{\square} = \square$

2. Use any values between 0–9 to make the equations true.

a. $3^{\square} \cdot 3^{\square} = 3^{\square}$

b. $3^{\square} \cdot 3^{\square} = 9^{\square}$

c. $3^{\square} \cdot 3^{\square} = \square^{\square}$

d. $(x^{\square})^{\square} = x^{\square}$

e. $(\square x^{\square})^{\square} = x^{\square}$

f. $\square^{\square} \cdot \square^{\square} = \square^{\square}$

g. $x^{\square} y^{\square} \cdot xy^{\square} = x^{\square} y^{\square}$

3. Use any values between -4 and 9 to make the equations true.

a. $$\frac{3^{\square} \cdot 3^{\square}}{3^{\square}} = 3^{\square}$$

b. $$(\square \cdot 10^{\square})\ (\square \cdot 10^{\square}) = \square \cdot 10^{\square}$$

c. $$\frac{(\square \cdot 10^{\square})}{(\square \cdot 10^{\square})} = \square \cdot 10^{\square}$$

Station F: Micro-Modeling with Pizza

A pizzeria on Planet Z serves two round pizzas of the same thickness in different sizes. The smaller one has a diameter of 30 cm and costs 30 zeds. The larger one has a diameter of 40 cm and costs 40 zeds. Which pizza is a better value for money? Show your reasoning.[1]

[1] Adapted from https://tasks.illustrativemathematics.org/

Station G: Studying Bacterial Growth

Mai and Tyler are lab partners studying bacterial growth. They were surprised to find that the population of the bacteria doubled every hour.[2]

1. The table shows that there were 2,000 bacteria at the beginning of the experiment. What was the size of the population of bacteria after 1 hour? After 2, 3, and 4 hours? Enter this information into the table:

Hours into study				0	1	2	3	4
Population (thousands)				2				

2. If you know the size of the population at a certain time, how do you find the population one hour later?

3. Mai said she thought that they could use the equation $P = 2t + 2$ to find the population at time t. Tyler said they thought that they could use the equation $P = 2 \cdot 2^t$. Decide whether either of these equations produces the correct populations for $t = 1, 2, 3, 4$.

4. Assuming the population doubled every hour before the study began, what was the population of the bacteria 1 hour *before* the students started their study? What about 3 hours before?

[2] Adapted from https://tasks.illustrativemathematics.org/

5. If you know the size of the population at a certain time, how do you find the population 1 hour *earlier*?

6. What number would you use to represent the time 1 hour before the study started? 2 hours before? 3 hours before? Finish filling in the table if you haven't already.

7. Now use Tyler's equation to find the population of the bacteria 1 hour before the study started. Use the equation to find the population of the bacteria 3 hours before. Do these values produce results consistent with the arithmetic you did earlier?

8. Use the context to explain why it makes sense that $2^{-n} = (\frac{1}{2})^n = \frac{1}{2^n}$. That is, describe why, based on the population growth, it makes sense to define 2 raised to a negative integer exponent as repeated multiplication by $\frac{1}{2}$.

Lesson 11: Exponential Situations as Functions

Learning Targets

- When I see relationships in descriptions, tables, equations, or graphs, I can determine whether the relationships are functions.
- I can use function notation to write equations that represent exponential relationships.

Bridge

1. The area, A, of circle C is given as a function of the radius, r, based on the rule $A = \pi r^2$. What is the area of circle C given radius:

   a. 5 inches?

   b. 12 inches?

2. The volume, V, of sphere S is given as a function of the radius, r, based on the rule $V = \frac{4}{3}\pi r^3$. What is the volume of sphere S given radius:

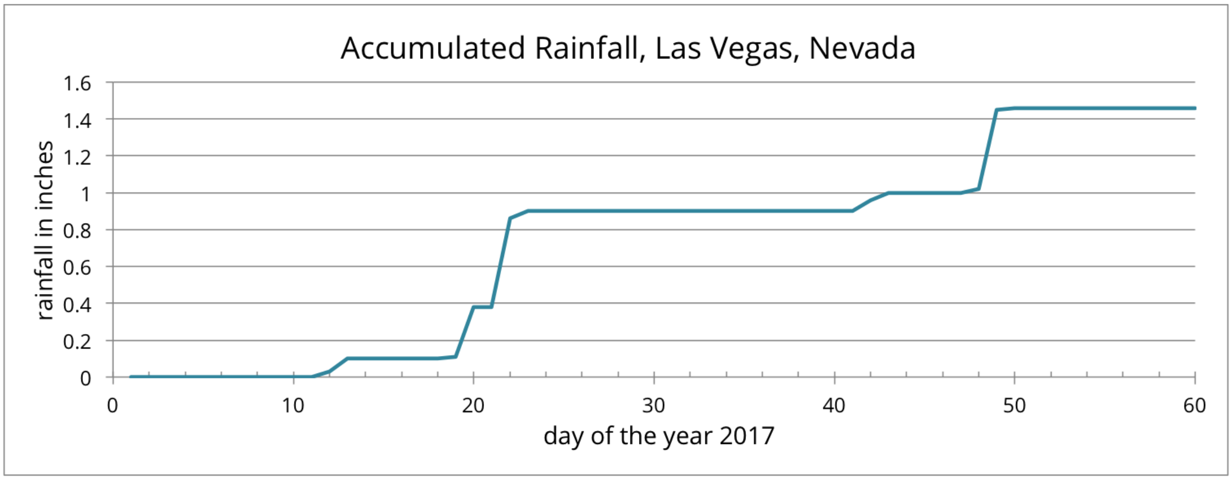
   a. 5 inches?

   b. 12 inches?

Warm-up: Rainfall in Las Vegas

Here is a graph of the accumulated rainfall in Las Vegas, Nevada, in the first 60 days of 2017.

Use the graph to support your answers to the following questions.



1. Is the accumulated amount of rainfall a function of time?

2. Is time a function of accumulated rainfall?

Activity 1: Moldy Bread

Clare noticed mold on the last slice of bread in a plastic bag. The area covered by the mold was about 1 square millimeter. She left the bread alone to see how the mold would grow. The next day, the area covered by the mold had doubled, and it doubled again the day after that.

1. If the doubling pattern continues, how many square millimeters will the mold cover 4 days after she noticed the mold? Show your reasoning.

2. Represent the relationship between the area A, in square millimeters, covered by the mold and the number of days d since the mold was spotted using:

    a. a table of values, showing the values from the day the mold was spotted through 5 days later
    b. an equation
    c. a graph

a.

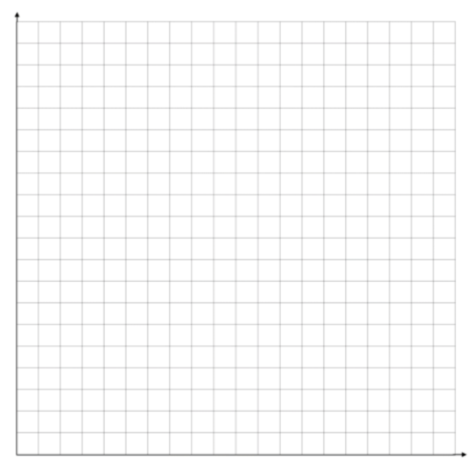
d	*A*

b.

Equation:

____

c.



3. Discuss with your partner: Is the relationship between the area covered by mold and the number of days a function? If so, write "_______ is a function of ________." If not, explain why it is not.

Are You Ready For More?

What do you think is an appropriate domain for the mold area function A? Explain your reasoning.

Activity 2: Functionally Speaking

Here are some situations we have seen previously. For each situation:

- Write a sentence of the form "____ is a function of _____."
- Indicate which is the independent and which is the dependent variable.
- Write an equation that represents the situation using function notation.

1. In a biology lab, a population of 50 bacteria reproduce by splitting. Every hour, on the hour, each bacterium splits into two bacteria.

2. Every year after a new car is purchased, it loses $\frac{1}{3}$ of its value. Let's say that the new car costs \$18,000.

3. In order to control an algae bloom in a lake, scientists introduce some treatment products. The day they begin treatment, the area covered by algae is 240 square yards. Each day since the treatment began, a third of the previous day's area (in square yards) remains covered by algae. Time t is measured in days.

Lesson Debrief

Lesson 11 Summary and Glossary

The situations we have looked at that are characterized by exponential change can be seen as functions. In each situation, there is a quantity (an independent variable) that determines another quantity (a dependent variable). They are functions because any value of the independent variable corresponds to one and only one value of the dependent variable. Functions that describe exponential change are called **exponential functions**.

Exponential function: A function that has a constant multiplier. Another way to say this is that it changes by equal factors over equal intervals. For example, $f(t) = 100,000 \cdot (\frac{1}{5})^t$ defines an exponential function. Any time t increases by 1, $f(t)$ is multiplied by a factor of $\frac{1}{5}$.

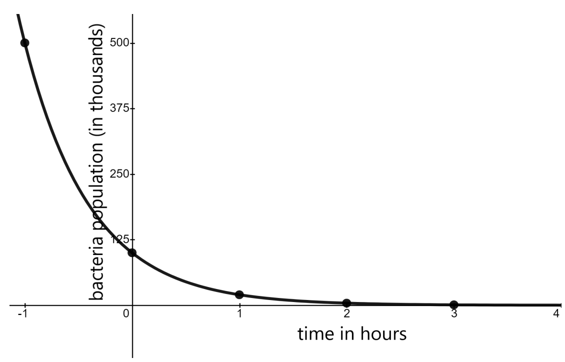
Functions can be represented by tables, graphs, equations, and descriptions.

For example, suppose t represents time in hours, and p is a bacteria population t hours after the bacteria population was measured. For each time t, there is only one value for the corresponding number of bacteria, so we can say that p is a function of t and we can write this as $p = f(t)$.

Table:

t	*f(t)*
-1	500,000
0	100,000
1	20,000
2	4,000
3	800

Graph:



Equation: $f(t) = 100,000 \cdot (\frac{1}{5})^t$

Notice the expression in the form of $a \cdot b^t$ (on the right side of the equation) is the same as in previous equations we wrote to represent situations characterized by exponential change.

Description:

There were 100,000 bacteria at the time the population was initially measured and the population decreases so that $\frac{1}{5}$ of it remains after each passing hour.

Unit 6 Lesson 11 Practice Problems

1. For an experiment, a scientist designs a can, 20 cm in height, that can hold water. A tube is installed at the bottom of the can allowing water to drain out. At the beginning of the experiment, the can is full. When the experiment starts, the water begins to drain, and the height of the water in the can decreases by a factor of $\frac{1}{3}$ each minute.

    a. Explain why the height of the water in the can is a function of time.

    b. The height, h, in cm, is a function f of time t in minutes since the beginning of the experiment, $h = f(t)$. Find an expression for $f(t)$.

    c. Find and record the values for f when t is 0, 1, 2, and 3.

    d. Find $f(4)$. What does $f(4)$ represent?

    e. Sketch a graph of f by hand or use graphing technology.

    f. What happens to the level of water in the can as time continues to elapse? How do you see this in the graph?

2. A scientist measures the height, h, of a tree each month, and m is the number of months since the scientist first measured the height of the tree.

   a. Is the height, h, a function of the month, m? Explain how you know.

   b. Is the month, m, a function of the height, h? Explain how you know.

3. A bacteria population is $10,000$. It triples each day.

   a. Explain why the bacteria population, b, is a function of the number of days, d, since it was measured to be $10,000$.

   b. Which variable is the independent variable in this situation?

   c. Write an equation relating b and d.

4.

a. Is the position, p, of the minute hand on a clock a function of the time, t?

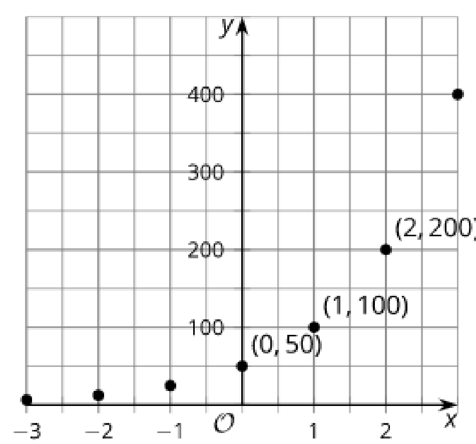
b. Is the time, t, a function of the position of the minute hand on a clock?

5. The area covered by a city is 20 square miles. The area grows by a factor of 1.1 each year since it was 20 square miles.

a. Explain why the area, a, covered by the city, in square miles, is a function of t, the number of years since its area was 20 square miles.

b. Write an equation for a in terms of t.

6. The graph shows an exponential relationship between x and y.



a. Write an equation representing this relationship.

b. What is the value of y when $x = -1$? Label this point on the graph.

c. What is the value of y when $x = -2$? Label this point on the graph.

(From Unit 6, Lesson 7)

7. Here are two expressions:

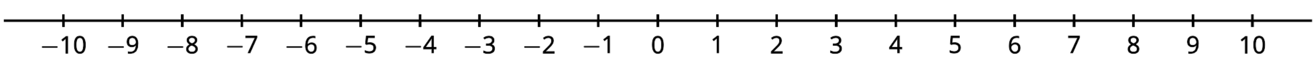
- $x^2 \cdot x^2$
- $(x^2)^2$

Is the value of the first expression greater than, less than, or equal to the value of the second expression? How do you know?

(From Unit 6, Lessons 1 and 2)

8. Here is an inequality: $3x + 1 > 34 - 4x$.

Graph the solution set to the inequality on the number line.

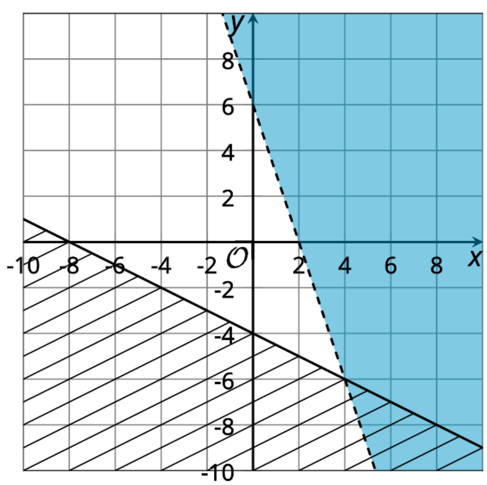


(From Unit 2)

9. Two inequalities are graphed on the same coordinate plane.

Select **all** of the points that are solutions to the system of the two inequalities.

- $(4, -6)$
- $(4, 6)$
- $(-4, -6)$
- $(-4, 6)$
- $(6, -8)$
- $(7, -9)$
- $(-8, 6)$



(From Unit 2)

10.

a. The distance around a circle, the circumference, C, is given as a function of the diameter, d, based on the rule $C = \pi d$. If a circle has a diameter of 6.5 inches, how long is the circumference?

b. The volume of a sphere, V, is given as a function of the radius, r, based on the rule $V = \frac{4}{3}\pi r^3$. If a sphere has a radius of 3.25 inches, what is the volume of the sphere?

(Addressing NC.7.G.4 and NC.8.F.9)

Lesson 12: Interpreting Exponential Functions

Learning Targets

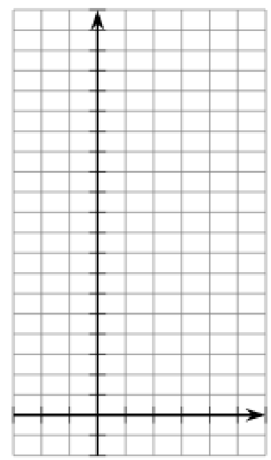
- I can analyze a situation and determine whether it makes sense to connect the points on the graph that represents the situation.
- When I see a graph of an exponential function, I can make sense of and describe the relationship using function notation.
- I can use graphing technology to graph exponential functions and analyze their domains.

Bridge

Here are descriptions of relationships between quantities.

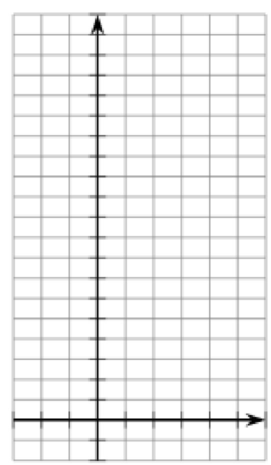
1. A cab charges $\$1.50$ per mile plus $\$3.50$ for entering the cab. The cost of the ride is a function of the miles, m, ridden and is defined by $c(m) = 1.50m + 3.50$.
    a. Make a table of at least five pairs of values that represent the relationship.
    b. Plot the points. Label the axes of the graph.
    c. Should the points be connected? Are there any input or output values that don't make sense? Explain.

m	c



2. The admission to the state park is $\$5.00$ per vehicle plus $\$1.50$ per passenger. The total admission for one vehicle is a function of the number of passengers, p, defined by the equation $a(p) = 5 + 1.50p$.
    a. Make a table of at least five pairs of values that represent the relationship.
    b. Plot the points. Label the axes of the graph.
    c. Should the points be connected? Are there any input or output values that don't make sense? Explain.

p	a



Warm-up: Equivalent or Not?

Lin and Diego are discussing two expressions: x^2 and 2^x.

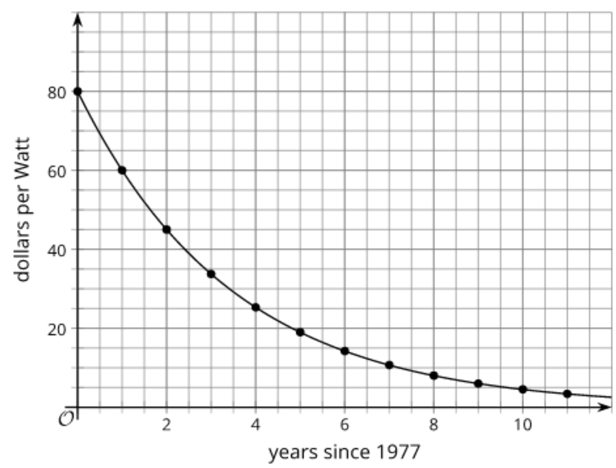
- Lin says, “I think the two expressions are equivalent.”
- Diego says, “I think the two expressions are only equal for *some* values of x.”

Do you agree with either of them? Explain or show your reasoning.

Activity 1: Cost of Solar Cells

The cost, in dollars, to install solar panels that produce 1 watt of solar power is a function of the number of years since 1977, t.

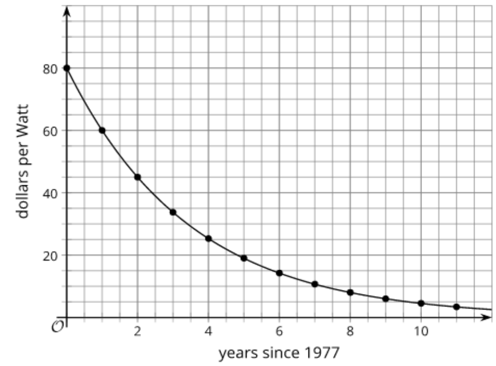
From 1977 to 1987, the cost could be modeled by an exponential function f. Here is the graph of the function.



1. What is the statement $f(9) \approx 6$ saying about this situation?

2. What is $f(4)$? What about $f(3.5)$? What do these values represent in this context?

3. When $f(t) = 45$, what is t? What does that value of t represent in this context?



4. By what factor did the cost of solar cells change each year? (If you get stuck, consider creating a table.)

5. How would you interpret the statement $f(-1) = 100$? Do you think this statement is reliable?

Activity 2: Paper Folding

1. The thickness, t, in millimeters, of a folded sheet of paper after it is folded n times is given by the function $t(n) = (0.05) \cdot 2^n$.

   a. What does the number 0.05 represent in the rule of the function?

   b. Use graphing technology to graph the function $t(n) = (0.05) \cdot 2^n$.

   c. How many folds does it take before the folded sheet of paper is more than 1 mm thick? How many folds before it is more than 1 cm thick? Explain how you know.

2. The area of a sheet of paper is 93.5 square inches.

    a. Find the remaining visible area of the sheet of paper after it is folded in half once, twice, and three times.

    b. Write a function rule expressing the visible area, a, of the sheet of paper in terms of the number of times it has been folded, n.

    c. Use graphing technology to graph the function.

    d. In this context, can n take negative values? Explain your reasoning.

    e. Can a take negative values? Explain your reasoning.

Are You Ready For More?

1. Using the model in this task, how many folds would be needed to get 1 meter in thickness? 1 kilometer in thickness?

2. Do some research: what is the current world record for the number of times humans were able to fold a sheet of paper?

Lesson Debrief

Lesson 12 Summary and Glossary

Earlier, we used equations to represent situations characterized by exponential change. For example, to describe the amount of caffeine c in a person's body t hours after an initial measurement of 100 mg, we used the equation $c = 100 \cdot (\frac{9}{10})^t$.

Notice that the amount of caffeine is a *function* of time, so another way to express this relationship is $c = f(t)$ where f is the function given by $f(t) = 100 \cdot (\frac{9}{10})^t$.

We can use this function to analyze the amount of caffeine. For example, when t is 3, the amount of caffeine in the body is $100 \cdot (\frac{9}{10})^3$ or $100 \cdot \frac{729}{1{,}000}$, which is 72.9. The statement $f(3) = 72.9$ means that 72.9 mg of caffeine are present 3 hours after the initial measurement.

We can also graph the function f to better understand what is happening. The point labeled P, for example, has coordinates approximately $(10, 35)$ so it takes about 10 hours after the initial measurement for the caffeine level to decrease to 35 mg.

A graph can also help us think about the values in the domain and range of a function. Because the body breaks down caffeine continuously over time, the domain of the function—the time in hours—can include non-whole numbers (for example, we can find the caffeine level when t is 3.5). In this situation, negative values for the domain would represent the time *before* the initial measurement. For example $f(-1)$ would represent the amount of caffeine in the person's body 1 hour before the initial measurement. The range of this function would not include negative values, as a negative amount of caffeine does not make sense in this situation.

Unit 6 Lesson 12 Practice Problems

1. The number of people with the flu during an epidemic is a function, f, of the number of days, d, since the epidemic began. The equation $f(d) = 50 \cdot (\frac{3}{2})^d$ defines f.

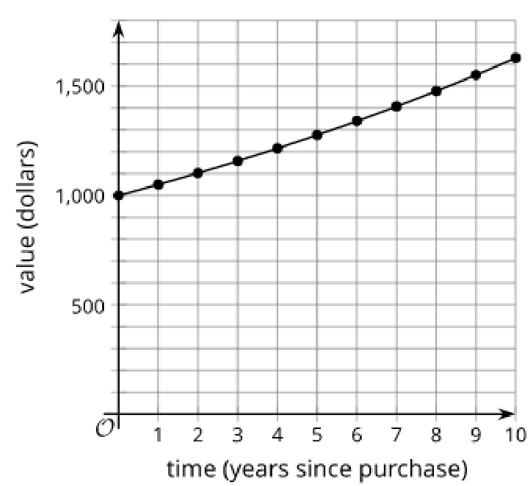
   a. How many people had the flu at the beginning of the epidemic? Explain how you know.

   b. How quickly is the flu spreading? Explain how you can tell from the equation.

   c. What does $f(1)$ mean in this situation?

   d. Does $f(3.5)$ make sense in this situation?

2. The function f gives the dollar value of a bond t years after the bond was purchased. The graph of f is shown.



   a. What is $f(0)$? What does it mean in this situation?

   b. What is $f(4.5)$? What does it mean in this situation?

   c. When is $f(t) = 1500$? What does this mean in this situation?

3. (*Technology required.*) A function f gives the number of stray cats in a town t years since the town started an animal control program. The program includes both sterilizing stray cats and finding homes to adopt them. An equation representing f is $f(t) = 243(\frac{1}{3})^t$.

   a. What is the value of $f(t)$ when t is 0? Explain what this value means in this situation.

   b. What is the approximate value of $f(t)$ when t is 0.5? Explain what this value means in this situation.

   c. What does the number $\frac{1}{3}$ tell you about the stray cat population?

   d. Use technology to graph f for values of t between 0 and 4. What graphing window allows you to see values of $f(t)$ that correspond to these values of t?

4. (*Technology required.*) Function g gives the amount of a chemical in a person's body, in milligrams, t hours since the patient took the drug. The equation $g(t) = 600 \cdot (\frac{3}{5})^t$ defines this function.

   a. What does the fraction $\frac{3}{5}$ mean in this situation?

   b. Use graphing technology to create a graph of g.

   c. What are the domain and range of g? Explain what they mean in this situation.

5. The dollar value of a moped is a function of the number of years, t, since the moped was purchased. The function f is defined by the equation $f(t) = 2,500 \cdot (\frac{1}{2})^t$.

   What is the **best** choice of domain for the function f?

   a. $-10 \leq t \leq 10$

   b. $-10 \leq t \leq 0$

   c. $0 \leq t \leq 10$

   d. $0 \leq t \leq 100$

6. All of the students in a classroom list their birthdays.

   a. Is the birthdate, b, a function of the student, s? Explain your reasoning.

   b. Is the student, s, a function of the birthdate, b? Explain your reasoning.

   (From Unit 6, Lesson 11)

7. The trees in a forest are suffering from a disease. The population of trees, p, in thousands, is modeled by the equation $p = 90 \cdot (\frac{3}{4})^t$, where t is the number of years since 2000.

   a. What was the tree population in 2001? What about in 1999?

   b. What does the number $\frac{3}{4}$ in the equation for p tell you about the population?

   c. What is the last year when the population was more than 250,000? Explain how you know.

   (From Unit 6, Lesson 7)

8. A patient receives 1,000 mg of a medicine. Each hour, $\frac{1}{5}$ of the medicine in the patient's body decays.

   a. Complete the table with the amount of medicine in the patient's body.

Hours since receiving medicine	**mg of medicine left in body**
0	
1	
2	
3	
h	

   b. Write an equation representing the number of mg of the medicine, m, in the patient's body h hours after receiving the medicine.

   c. Use your equation to find m when $h = 10$. What does this mean in terms of the medicine?

(From Unit 6, Lesson 6)

9. Rewrite each expression using the fewest number of exponents.

   a. $3x^a \cdot x^b$

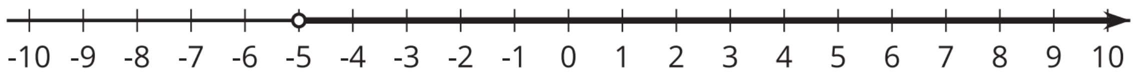
   b. $\frac{-5y^5 \cdot 4y^{-2} \cdot y^4}{y^{-2}}$

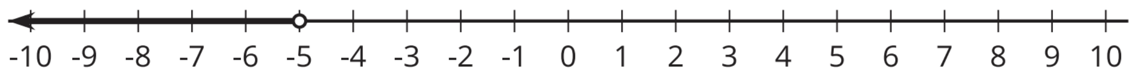
   c. $\frac{(5n^2)^4}{-5n^2}$

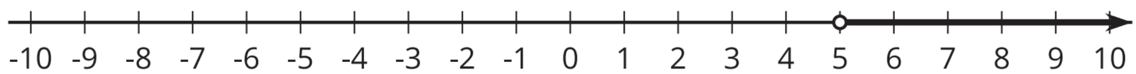
(From Unit 6, Lesson 2)

10. Mai wants to graph the solution to the inequality $5x-4>2x-19$ on a number line. She solves the equation $5x-4=2x-19$ for x and gets $x=-5$.

Which graph shows the solution to the inequality?

a. 

b. 

c. 

d. 

(From Unit 2)

11. A car is traveling on a small highway and is either going 55 miles per hour or 35 miles per hour, depending on the speed limits, until it reaches its destination 200 miles away. Letting x represent the amount of time in hours that the car is going 55 miles per hour, and y being the time in hours that the car is going 35 miles per hour, an equation describing the relationship is: $55x+35y=200$.

    a. If the car spends 2.5 hours going 35 miles per hour on the trip, how long does it spend going 55 miles per hour?

    b. If the car spends 3 hours going 55 miles per hour on the trip, how long does it spend going 35 miles per hour?

    c. If the car spends no time going 35 miles per hour, how long would the trip take? Explain your reasoning.

(Addressing NC.8.F.1 and NC.8.F.5)[1]

[1] Adapted from IM 6–8 Math https://curriculum.illustrativemathematics.org/MS/index.html, which was originally developed by Open Up Resources and authored by Illustrative Mathematics, and is copyright 2017–2019 by Open Up Resources. It is licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0). OUR's 6–8 Math Curriculum is available at https://openupresources.org/math-curriculum/. Adaptations and updates to IM 6–8 Math are copyright 2019 by Illustrative Mathematics, and are licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0).

Lesson 13: Modeling Exponential Behavior

Learning Targets

- When given data, I can determine an appropriate model for the situation described by the data.
- I can use exponential functions to model situations that involve exponential growth or decay.

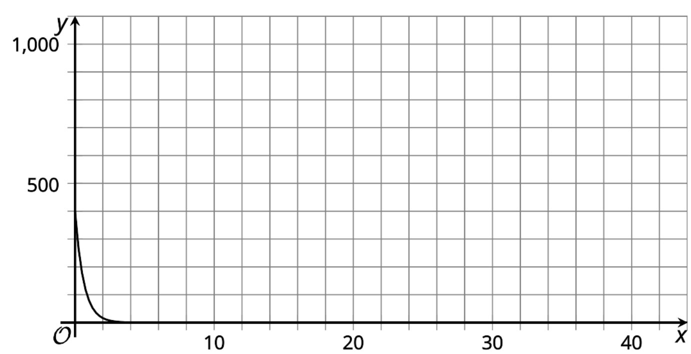
Bridge

Order these three values from least to greatest. Explain or show your reasoning.[1]

- 65% of 80
- 82% of 50
- 170% of 30

Warm-up: Wondering about Windows

Here is a graph of a function f defined by $f(x) = 400 \cdot (0.2)^x$.



1. Identify the approximate graphing window shown.

2. Suggest a new graphing window that would:
   a. make the graph more informative or meaningful
   b. make the graph less informative or meaningful

   Be prepared to explain your reasoning.

[1] Adapted from IM 6–8 Math https://curriculum.illustrativemathematics.org/MS/index.html, which was originally developed by Open Up Resources and authored by Illustrative Mathematics, and is copyright 2017–2019 by Open Up Resources. It is licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0). OUR's 6–8 Math Curriculum is available at https://openupresources.org/math-curriculum/. Adaptations and updates to IM 6–8 Math are copyright 2019 by Illustrative Mathematics, and are licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0).

Activity 1: Beholding Bounces

Here are measurements for the maximum height of a tennis ball after bouncing several times on a hard surface.

n, bounce number	h, height (centimeters)
0	75
1	40
2	21.5
3	10
4	5.5

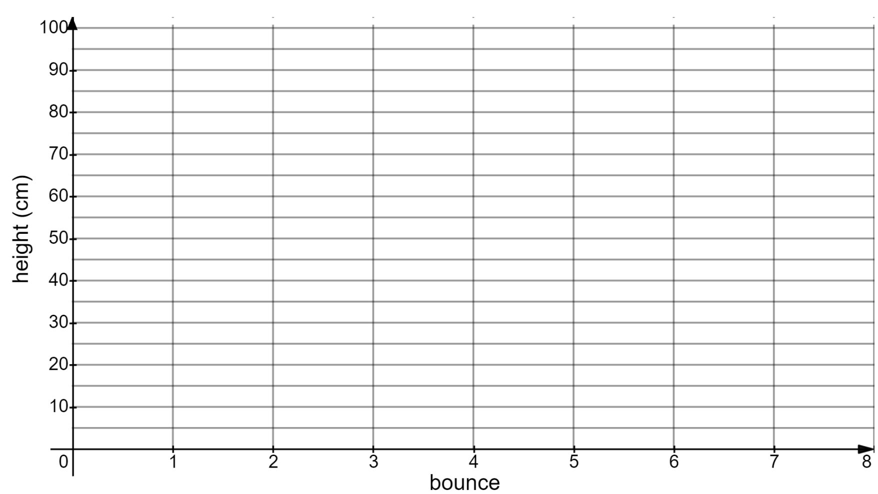
1. Which is more appropriate for modeling the maximum height h, in centimeters, of the tennis ball after n bounces: a linear function or an exponential function? Use data from the table to support your answer.

2. Regulations say that a tennis ball dropped on a hard surface should rebound to a height between 53% and 58% of the height from which it is dropped. Does the tennis ball here meet this requirement? Explain your reasoning.

3.
   a. Remember that coordinate axes are usually called the x- and y-axes. We usually label coordinates as (x, y). The variables used for our data are labeled n and h. Which axes do the n and the h represent?

   b. Graph the data on the coordinate grid.

   c. Describe the shape and relationship of the data.



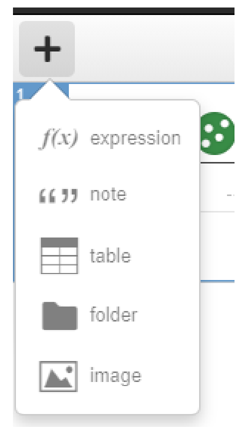
4. Now you are going to graph the data in Desmos and calculate the exponential regression for the model. Follow the instructions in the box below to complete this step.

a. Access desmos.com/calculator.

b. Enter your data into a table. To access a table, click on the plus sign at the top left, and select the table option.

c. Select an appropriate window for your data.

d. In your second line, type in $y_1 \sim ab^{x_1}$. Select "Log Mode" when the option appears.



5. Write the exponential regression equation, given in Desmos after completing the boxed instructions above, that models the bounce height, h, after n bounces for this tennis ball.

6. About how many bounces will it take before the rebound height of the tennis ball is less than 1 centimeter? Explain your reasoning.

Activity 2: Beholding More Bounces

The table shows some heights of a ball after a certain number of bounces.

Bounce number	Height in centimeters
0	
1	
2	73.5
3	51.5
4	36

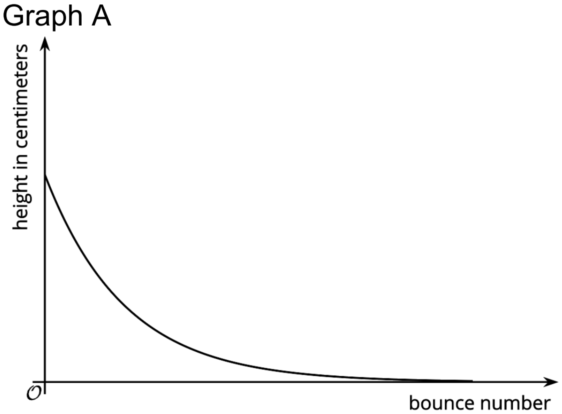
1. Is this ball more or less bouncy than the tennis ball in the earlier task? Explain or show your reasoning.

2. From what height was the ball dropped? Explain or show your reasoning.

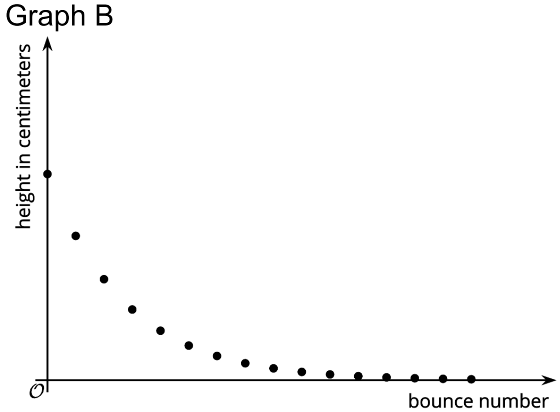
3. Use technology to determine an exponential regression equation that represents the bounce height of the ball, h, in centimeters after n bounces.

4. Which graph would more appropriately represent the equation for h: graph A or graph B? Explain your reasoning.

Graph A



Graph B



5. Will the n-th bounce of this ball ever be lower than the n-th bounce of the tennis ball? Explain your reasoning.

Activity 3: Which is the Bounciest of All?

Your teacher will give your group three different kinds of balls.

Your goal is to measure the rebound heights, model the relationship between the number of bounces and the heights, and compare the bounciness of the balls.

1. Complete the table. Make sure to note which ball goes with which column.

n, number of bounces	a, height for ball 1 (cm)	b, height for ball 2 (cm)	c, height for ball 3 (cm)
0			
1			
2			
3			
4			

2. Which one appears to be the bounciest? Which one appears to be the least bouncy? Explain your reasoning.

3. For each one, write an equation expressing the bounce height in terms of the bounce number, n.

Ball 1: Ball 2: Ball 3:

4. Explain how the equations could tell us which one is the most bouncy.

5. If the bounciest one were dropped from a height of 300 cm, what equation would model its bounce height, h?

Lesson Debrief

Lesson 13 Summary and Glossary

Sometimes data suggest an exponential relationship. For example, this table shows the bounce heights of a certain ball. We can see that the height decreases with each bounce.

To find out what fraction of the height remains after each bounce, we can divide two consecutive values: $\frac{61}{95}$ is about 0.642, $\frac{39}{61}$ is about 0.639, and $\frac{26}{39}$ is about 0.667.

All of these quotients are close to $\frac{2}{3}$. This suggests that there is an exponential relationship between the number of bounces and the height of the bounce, and that the height is decreasing with a factor of about $\frac{2}{3}$ for each successive bounce.

Bounce number	Bounce height in centimeters
1	95
2	61
3	39
4	26

Note that $\frac{2}{3}$ is an estimate of the rebound factor. In fact, the true factor may be 0.65, 0.6672, or something else. It's also possible that the relationship is very close to exponential but not perfect. For this reason, when measurement error is a possibility we do not choose to use very specific decay factors like 0.6672, since we are estimating anyway.

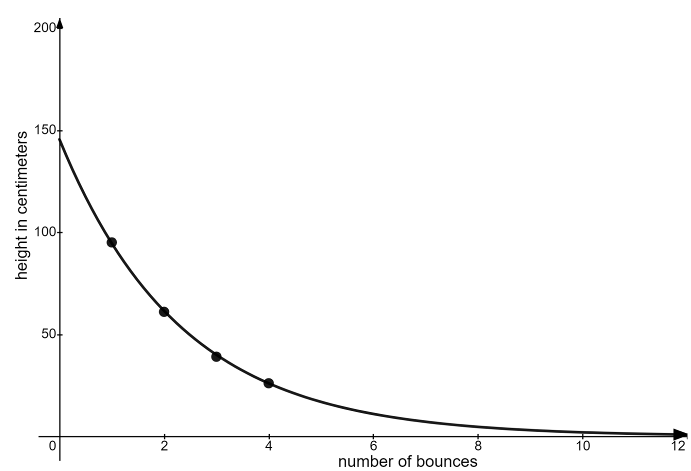
Using technology, such as Desmos, is an efficient way to determine a model that closely aligns to the provided measurements. In order to perform exponential regression, input the data provided in the table into desmos. Then type y1~ab^x1 and select "Log Mode." Desmos will then give you the values of a and b to use for an exponential function that best models the given measurements.

Now we are ready to write an equation that models the height, h, of the ball, in cm, after n bounces:

$$h = 145.5 \cdot (0.65)^n$$

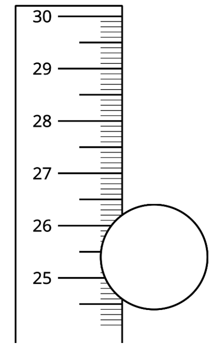
Here is a graph of the equation:

This graph shows both the points from the data and the points generated by the equation, which can give us new insights.



Unit 6 Lesson 13 Practice Problems

1. Here is an image showing the highest point of the path of a ball after one bounce.



   Someone is collecting data to model the bounce height of this ball after each bounce. Which measurement for the location of the top of the ball would be the **best** one to record?

   a. 26 cm

   b. 26.4 cm

   c. 26.43 cm

   d. 26.431 cm

2. Function h describes the height of a ball, in inches, after n bounces and is defined by the equation $h(n) = 120 \cdot (\frac{4}{5})^n$.

   a. What is $h(3)$? What does it represent in this situation?

   b. Could $h(n)$ be 150? Explain how you know.

   c. Which ball loses its height more quickly, this ball or a tennis ball whose height in inches after n bounces is modeled by the function f where $f(n) = 50 \cdot (\frac{5}{9})^n$?

   d. How many bounces would it take before the ball modeled by function h bounces less than 12 inches from the surface?

3. After its second bounce, a ball reached a height of 80 cm. The rebound factor for the ball was 0.7. From approximately what height, in cm, was the ball dropped?

   a. 34

   b. 49

   c. 115

   d. 163

4. Which equation is most appropriate for modeling this data?

x	1	2	3	4	5	6
y	79	101	124	158	195	244

   a. $y = 64 \cdot (1.25)^x$

   b. $y = 79 \cdot (1.25)^x$

   c. $y = 79 + 1.25x$

   d. $y = 64 + 22x$

5. The table shows the number of employees and number of active customer accounts for some different marketing companies.

Number of employees	1	2	3	4	10
Number of customers	4	8	13	17	39

   Would a linear or exponential model for the relationship between number of employees and number of customers be more appropriate? Explain how you know.

6. A bank account has a balance of 1,000 dollars. It grows by a factor of 1.04 each year.

   a. Explain why the balance, in dollars, is a function, f, of the number of years, t, since the account was opened.

   b. Write an equation defining f.

(From Unit 6, Lesson 11)

7. Rewrite each expression with the least number of exponents (all positive).[2]

   a. $\frac{x^5y^{12}z^0}{x^8y^9}$

   b. $\frac{(y^a)^c}{y^b}$

   c. $\frac{r^5s^3}{rs^3}$

(From Unit 6, Lesson 2)

[2] Adapted from Open Up Resources. Access the full curriculum, supporting tools, and educator communities at openupresources.org.

8. The table shows the number of people, n, who went to see a musical on the d^{th} day of April.

   a. What is the average rate of change for the number of people from day 1 to day 7?

   b. Is the average rate of change a good measure for how the number of people changed throughout the week? Explain your reasoning.

d	n
1	1,534
2	2,324
3	2,418
4	2,281
5	2,350
6	2,394
7	1,720

(From Unit 5)

Lesson 14: Reasoning about Exponential Graphs (Part One)

Learning Targets

- I can describe the effect of changing a and b on a graph that represents $f(x) = a \cdot b^x$.
- I can use equations and graphs to compare exponential functions.

Bridge

When The Cookie Kit opened their bakery, they used the function $c(x) = 2.50x + 6.50$ to determine the amount to charge for a custom made-to-order cookie order of x dozen of cookies.

However, due to supply chain problems, the bakery was forced to increase the prices. The function $d(x) = 3.25x + 6.50$ is the function the Cookie Kit now uses for custom orders.

1. How do the functions c and d compare?

2. How do the graphs of c and d compare?

Warm-up: Spending Gift Money

Jada received a gift of $180. In the first week, she spent a third of the gift money. She continues spending a third of what is left each week thereafter. Which equation best represents the amount of gift money g, in dollars, she has after t weeks? Be prepared to explain your reasoning.

a. $g = 180 - \frac{1}{3}t$

b. $g = 180 \cdot \left(\frac{1}{3}\right)^t$

c. $g = \frac{1}{3} \cdot 180^t$

d. $g = 180 \cdot \left(\frac{2}{3}\right)^t$

Activity 1: Equations and Their Graphs

1. Each of the following functions, f, g, h, and j, represents the value of an investment in a different company, in dollars, as a function of time x, in years. They are each written in the form $m(x) = a \cdot b^x$.

   $f(x) = 50 \cdot 2^x$

   $g(x) = 50 \cdot 3^x$

   $h(x) = 50 \cdot (\frac{3}{2})^x$

   $j(x) = 50 \cdot (0.5)^x$

   a. Use graphing technology to graph each function on the same coordinate plane.

   b. Explain how changing the value of b changes the graph.

2. Here are equations defining functions p, q, and r. They are also written in the form $m(x) = a \cdot b^x$.

   $p(x) = 10 \cdot 4^x$

   $q(x) = 40 \cdot 4^x$

   $r(x) = 100 \cdot 4^x$

   a. Use graphing technology to graph each function and check your prediction.

   b. Explain how changing the value of a changes the graph.

Are You Ready For More?

Consider stock investments whose balances are given by the following functions:

a. $f(x) = 10 \cdot 3^x$

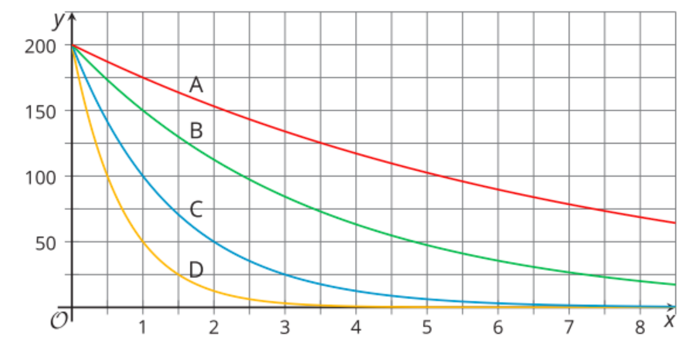
b. $g(x) = 3^{x+2}$

c. $h(x) = \frac{1}{2} \cdot 3^{x+3}$

Which stock is doing the best? Does your choice depend on x?

Activity 2: Graphs Representing Exponential Decay

1. $m(x) = 200 \cdot (\frac{1}{4})^x$
2. $n(x) = 200 \cdot (\frac{1}{2})^x$
3. $p(x) = 200 \cdot (\frac{3}{4})^x$
4. $q(x) = 200 \cdot (\frac{7}{8})^x$



1. Match each equation with a graph. Be prepared to explain your reasoning.

2. Functions f and g are defined by these two equations: $f(x) = 1,000 \cdot (\frac{1}{10})^x$ and $g(x) = 1,000 \cdot (\frac{9}{10})^x$.

   a. Which function is decaying more quickly? Explain your reasoning.

   b. Use graphing technology to verify your response.

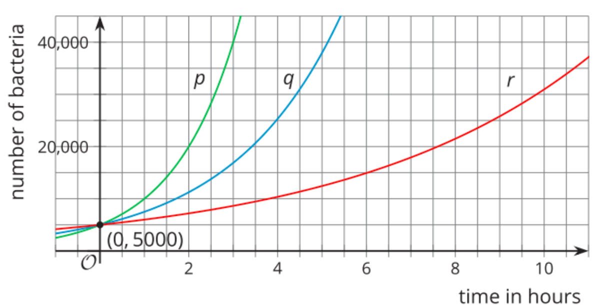
Lesson Debrief

Lesson 14 Summary and Glossary

An exponential function can give us information about a graph that represents it.

For example, suppose the function q represents a bacteria population t hours after it is first measured and $q(t) = 5,000 \cdot (1.5)^t$. The number 5,000 is the bacteria population measured when t is 0. The number 1.5 indicates that the bacteria population increases by a factor of 1.5 each hour.

A graph can help us see how the starting population (5,000) and growth factor (1.5) influence the population. Suppose functions p and r represent two other bacteria populations and are given by $p(t) = 5,000 \cdot 2^t$ and $r(t) = 5,000 \cdot (1.2)^t$. Here are the graphs of p, q, and r.



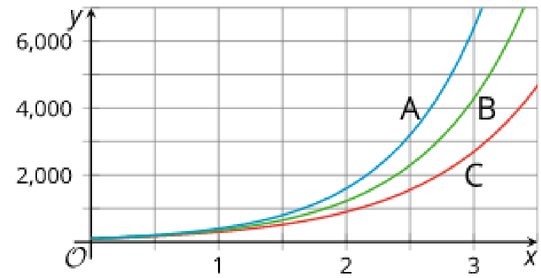
All three populations have an initial value of $5,000$, but the graph of r grows more slowly than the graph of q while the graph of p grows more quickly. This makes sense because a population that doubles every hour is growing more quickly than one that increases by a factor of 1.5 each hour, and both grow more quickly than a population that increases by a factor of 1.2 each hour.

The graph also helps us see the **end behavior** of the functions describing the bacteria populations. As time goes on, all of the functions produce larger and larger outputs. They are unlike exponential decay functions, which produce outputs closer and closer to zero as the inputs get larger.

End behavior of a function: The characteristics that a function eventually takes on when its input becomes large enough. This might be that the outputs approach a certain value or that they grow without bound. We can also think about end behavior when comparing eventual rates of change of two functions.

Unit 6 Lesson 14 Practice Problems

1. Here are equations defining three exponential functions, f, g, and h.



$f(x) = 100 \cdot 3^x$

$g(x) = 100 \cdot (3.5)^x$

$h(x) = 100 \cdot 4^x$

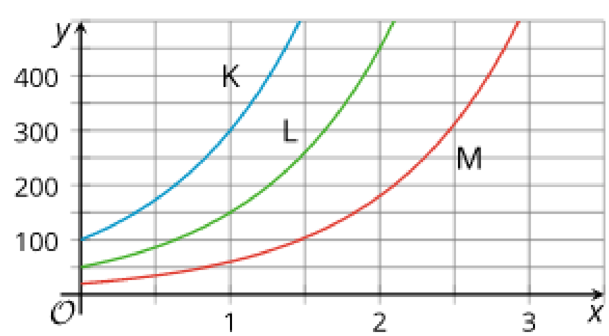
a. Which of these functions grows the least quickly? Which one grows the most quickly? Explain how you know.

b. The three given graphs represent f, g, and h. Which graph corresponds to each function?

c. Why do all three graphs share the same intersection point with the vertical axis?

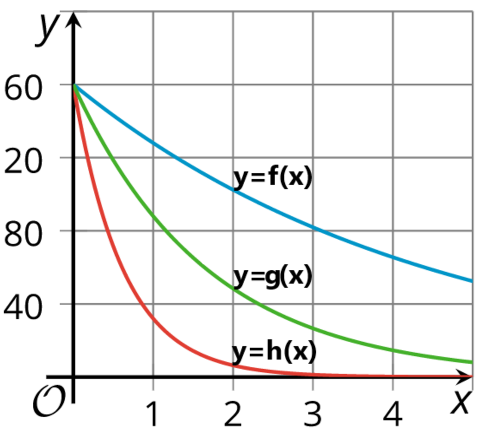
2. Here are graphs of three exponential equations. Match each equation with its graph.

a. $y = 20 \cdot 3^x$	1. Graph K
b. $y = 50 \cdot 3^x$	2. Graph L
c. $y = 100 \cdot 3^x$	3. Graph M



3. The function f is given by $f(x) = 160 \cdot (\frac{4}{5})^x$ and the function h is given by $h(x) = 160 \cdot (\frac{1}{5})^x$.

   If function g is defined by $g(x) = a \cdot b^x$, what can you say about a and b? Explain your reasoning.

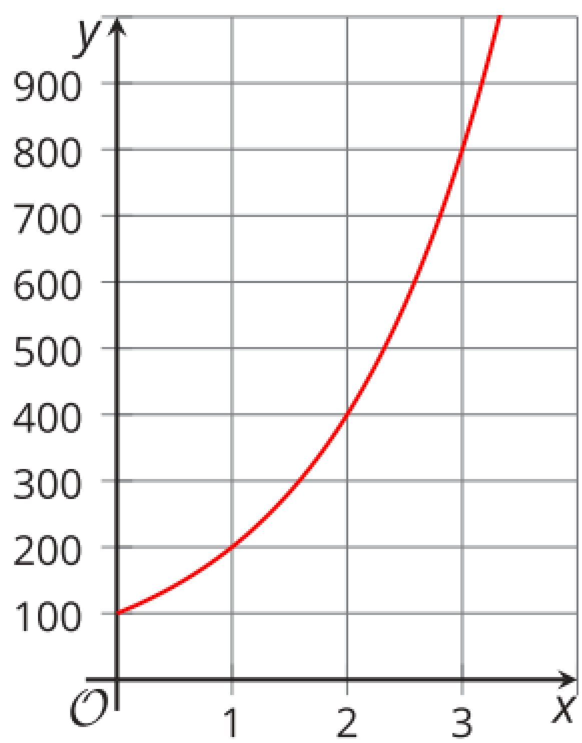


4. Here is a graph of $y = 100 \cdot 2^x$.

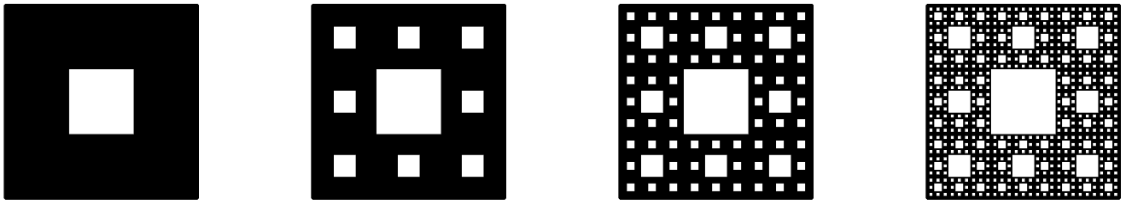
   On the same coordinate plane:

   a. Sketch a graph of $y = 50 \cdot 2^x$ and label it A.

   b. Sketch a graph of $y = 200 \cdot 2^x$ and label it B.



5. (*Technology required.*) Start with a square with area 1 square unit (not shown). Subdivide it into nine squares of equal area and remove the middle one to get the first figure shown.



a. What is the area of the first figure shown?

b. Take the remaining eight squares, subdivide each into nine equal squares, and remove the middle one from each. What is the area of the figure now?

c. Continue the process and find the area for stages 3 and 4.

d. Write an equation representing the area A at stage n.

e. Use technology to graph your equation.

f. Use your graph to find the first stage when the area is less than $\frac{1}{2}$ square unit.

(From Unit 6, Lesson 7)

6.

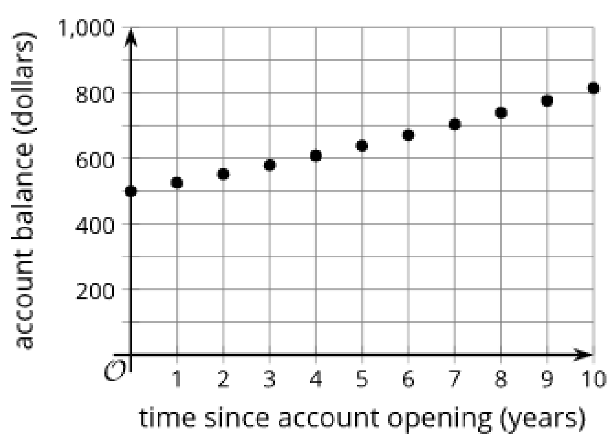
a. Give a positive value of x that would make the inequality $(x^3)^4 > (x^3)^{-4}$ true?

b. Give a positive value of x that would make the inequality $(x^3)^4 < (x^3)^{-4}$ true?

c. Give a positive value of x that would make the equation $(x^3)^4 = (x^3)^{-4}$ true?

(From Unit 6, Lessons 1 and 2)

7. The equation $b = 500 \cdot (1.05)^t$ gives the balance of a bank account t years since the account was opened. The graph shows the annual account balance for 10 years.

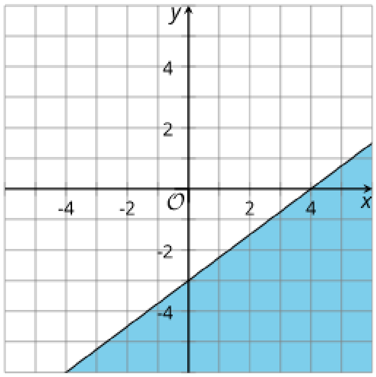


a. What is the average rate of change of the account balance over the 10 years?

b. Is the average rate of change a good measure of how the bank account balance varies? Explain your reasoning.

(From Unit 5)

8. Choose the inequality whose solution region is represented by this graph.



   a. $3x - 4y > 12$

   b. $3x - 4y \geq 12$

   c. $3x - 4y < 12$

   d. $3x - 4y \leq 12$

(From Unit 2)

9. During cross country season, Priya and Andre each set aside $\$20.00$ to spend on snacks after practice. Priya buys a Gatorade each day after practice, and Andre buys a granola bar each day after practice. The function $p(x) = -1.25x + 20$ represents the amount of money Priya has left after x days, and the function $a(x) = -0.85x + 20$ represents the amount of money Andre has left after x days.

   a. How do the functions p and a compare?

   b. How do the graphs of p and a compare?

   c. What do 1.25 and 0.85 represent in the functions?

(Addressing NC.8.F.2 and NC.8.F.4)

Lesson 15: Reasoning about Exponential Graphs (Part Two)

Learning Targets

- When I know two points on a graph of an exponential function, I can write an equation for the function.
- I can explain the meaning of the intersection of the graphs of two functions in terms of the situations they represent.

Bridge

Each function represents the amount in different bank accounts after t weeks.

$A(t) = 500$

$B(t) = 500 + 40t$

$C(t) = 500 - 40t$

$D(t) = 500(1.5)^t$

$E(t) = 500(0.75)^t$

Use technology to create a graph of each function. Describe in words how the money in each account is changing week by week. How do you see this in the graphs?

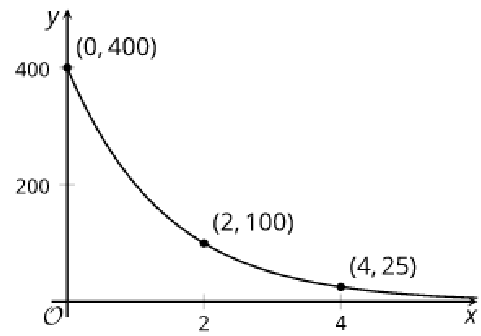
Warm-up: Four Functions

Which one doesn't belong? Explain your reasoning.

a. $f(n) = 8 \cdot 2^n$	b. $g(n) = 2 \cdot 8^n$
c. $h(n) = 8 + 2n$	d. $j(n) = 8 \cdot \left(\frac{1}{2}\right)^n$

Activity 1: Value of A Computer

1. Here is a graph representing an exponential function f. The function f gives the value of a computer, in dollars, as a function of time, x, measured in years since the time of purchase.



   Based on the graph, what can you say about the following?

   a. The purchase price of the computer

   b. The value of f when x is 1

   c. The meaning of $f(1)$

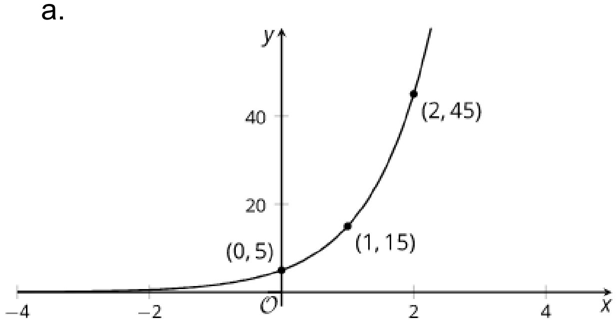
   d. How the value of the computer is changing each year

   e. An equation that defines f

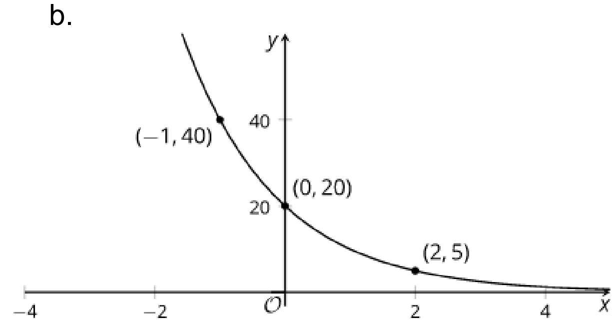
   f. Whether the value of f will reach 0 after 10 years

2. Here are graphs of two exponential functions. For each, write an equation that defines the function and find the value of the function when x is 5.

a.



b.



Are You Ready For More?

Consider a function f defined by $f(x) = a \cdot b^x$.

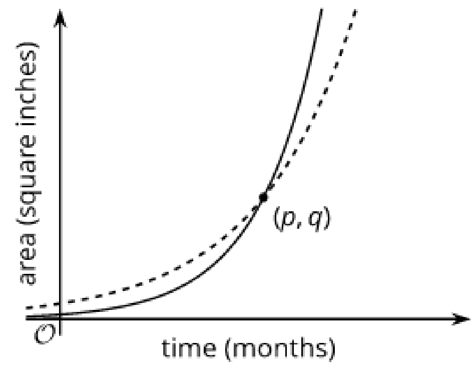
1. If the graph of f goes through the points $(2, 10)$ and $(8, 30)$, would you expect $f(5)$ to be less than, equal to, or greater than 20?

2. If the graph of f goes through the points $(2, 30)$ and $(8, 10)$, would you expect $f(5)$ to be less than, equal to, or greater than 20?

Activity 2: Moldy Wall

Here are graphs representing two functions and descriptions of two functions.

- Function f: The area of a wall that is covered by mold A, in square inches, doubling every month.
- Function g: The area of a wall that is covered by mold B, in square inches, tripling every month.



1. Which graph represents each function? Label the graphs accordingly and explain your reasoning.

2. When the mold was first spotted and measured, was there more of mold A or mold B? Explain how you know.

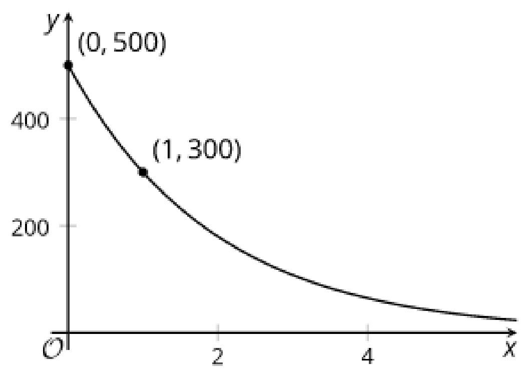
3. What does the point (p, q) tell us in this situation?

Lesson Debrief

Lesson 15 Summary and Glossary

If we have enough information about a graph representing an exponential function f, we can write a corresponding equation. Here is a graph of $y = f(x)$.

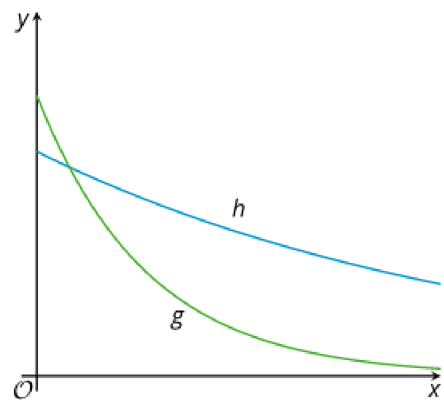
An equation defining an exponential function has the form $f(x) = a \cdot b^x$. The value of a is the starting value or $f(0)$, so it is the y-intercept of the graph. We can see that $f(0)$ is 500 and that the function is decreasing.



The value of b is the growth or decay factor: in this case, decay. It is the number by which we multiply the function's output at x to get the output at $x+1$. To find the decay factor for f, we can calculate $\frac{f(1)}{f(0)}$, which is $\frac{300}{500}$ or $\frac{3}{5}$. So an equation that defines f is:

$$f(x) = 500 \cdot (\tfrac{3}{5})^x$$

We can also use graphs to compare functions. Here are graphs representing two different exponential functions, labeled g and h. Each one represents the area of algae (in square meters) in a pond, x days after certain fish were introduced.



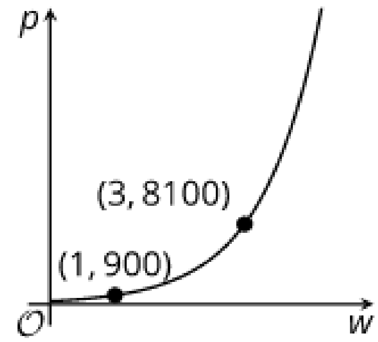
- Pond A had 40 square meters of algae. Each day, its area shrinks to $\frac{8}{10}$ of the area on the previous day.
- Pond B had 50 square meters of algae. Each day, its area shrinks to $\frac{2}{5}$ of the area on the previous day.

Can you tell which graph corresponds to which algae population?

We can see that the y-intercept of g's graph is greater than the y-intercept of h's graph. We can also see that g has a smaller decay factor than h because as x increases by the same amount, g is retaining a smaller fraction of its value compared to h. This suggests that g corresponds to pond B and h corresponds to pond A.

Unit 6 Lesson 15 Practice Problems

1. Here is a graph of p, an insect population, w weeks after it was first measured. The population grows exponentially.

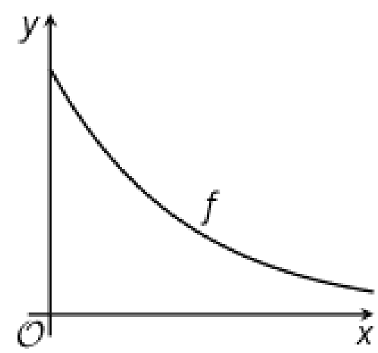


   a. What is the weekly growth factor for the insect population?

   b. What was the population when it was first measured?

   c. Write an equation relating p and w.

2. Here is a graph of the function f defined by $f(x) = a \cdot b^x$.



   Select **all** possible values of b.

   a. 0

   b. $\frac{1}{10}$

   c. $\frac{1}{2}$

   d. $\frac{9}{10}$

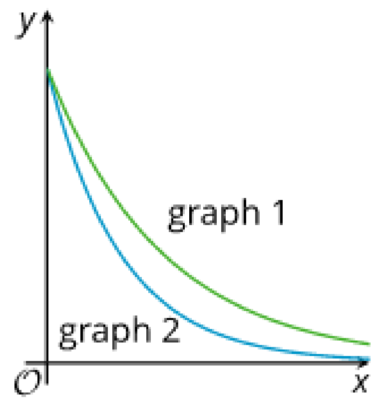
   e. 1

   f. 1.3

   g. $\frac{18}{5}$

3. The function f is given by $f(x) = 50 \cdot (\frac{1}{2})^x$, and the function g is given by $g(x) = 50 \cdot (\frac{1}{3})^x$.

   Here are graphs of f and g.

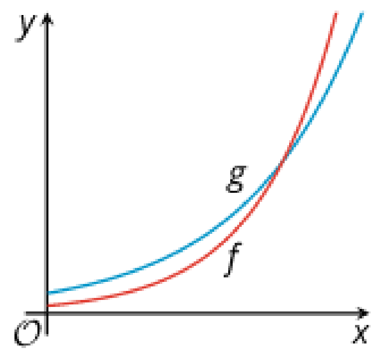
   

   Kiran says that since $3 > 2$, the graph of g lies above the graph of f so graph 1 is the graph of g and graph 2 is the graph of f.

   Do you agree? Explain your reasoning.

4. The function f is defined by $f(x) = 50 \cdot 3^x$. The function g is defined by $g(x) = a \cdot b^x$.

   Here are graphs of f and g.

   a. How does a compare to 50? Explain how you know.

   b. How does b compare to 3? Explain how you know.

5. A ball was dropped from a height of 150 cm. The rebound factor of the ball is 0.8. About how high, in centimeters, did the ball go after the third bounce?

   a. 77

   b. 96

   c. 234

   d. 293

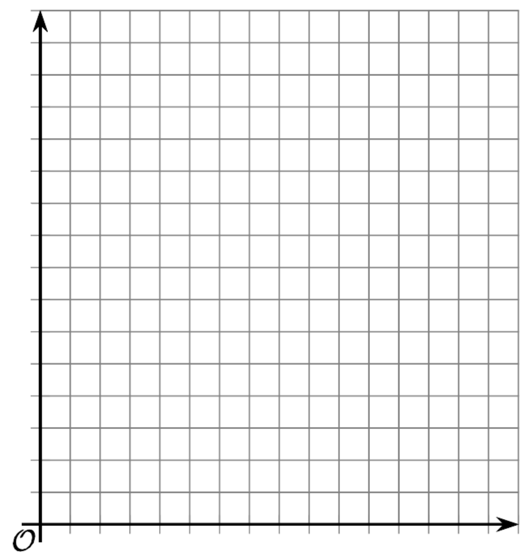
(From Unit 6, Lesson 13)

6. The dollar value of a car is a function, f, of the number of years, t, since the car was purchased. The function is defined by the equation $f(t) = 12,000 \cdot (\frac{3}{4})^t$.

   a. How much was the car worth when it was purchased? Explain how you know.

   b. What is $f(2)$? What does this tell you about the car?

   c. Sketch a graph of the function f.



   d. About when was the car worth $6,000? Explain how you know.

(From Unit 6, Lesson 12)

7. (*Technology required.*) The equation $y = 600,000 \cdot (1.055)^t$ represents the population of a country t decades after the year 2000.

   Use graphing technology to graph the equation. Then, set the graphing window so that you can simultaneously see points on the graph representing the population predicted by the model in 1980 and in the year 2020. What graphing window did you use?

   (From Unit 6, Lesson 7)

8. Solve each:

   a. $\frac{3x^{-9}}{6x^{-11}}$

   b. $\frac{a^2b^{-3}}{a^2b^{-1}}$

   c. $(\frac{2x}{y})^{-5}$

   d. $(\frac{c^4d}{c^{-2}})^{-4}$

   (From Unit 6, Lessons 1 and 2)

9. A triathlon athlete runs at an average rate of 8.2 miles per hour, swims at an average rate of 2.4 miles per hour, and bikes at a rate of 16.1 miles per hour. At the end of one training session (during which she did not run), she swam and biked more than 20 miles in total.

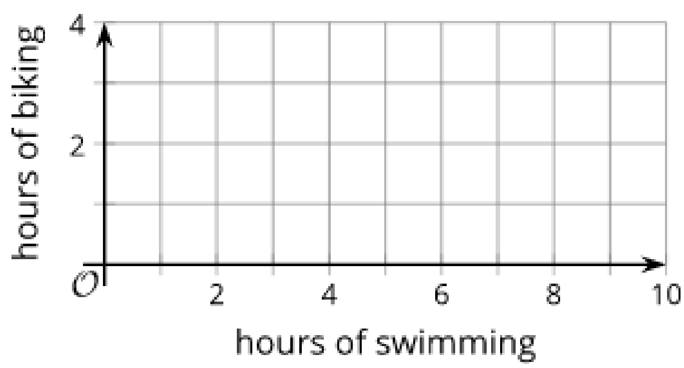
   a. Is it possible that she swam and biked for the following amounts of time in that session? Show your reasoning.

      i. Swam for 0.5 hour and biked 1.25 hours.

      ii. Swam for $\frac{1}{3}$ hour and biked for 70 minutes.

   b. Write an inequality to represent the relationship between the time she swam and biked, in hours, and the total distance she traveled. Be sure to specify what each variable represents.

   c. Use your inequality to graph a solution set that represents all the possible combinations of swimming and running times that meet the distance constraint (regardless of whether the times are realistic).



(From Unit 3)

10. (*Technology required.*) The population of five different cities after x years is described by the following functions, in thousands of people:

- City R: $R(x) = 200$
- City S: $S(x) = 200(1.1)^x$
- City T: $T(x) = 200 + 1.1x$
- City U: $U(x) = 200(0.9)^x$
- City V = $V(x) = 200 - 1.1x$

Use technology to create a graph of each function. Describe in words how the population of each city is changing year by year. How is this represented in the graph?

(Addressing NC.8.F.4 and NC.8.F.5)

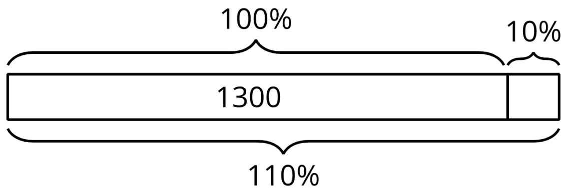
Lesson 16: Functions Involving Percent Change

Learning Targets

- I can calculate the growth or decay factor given the percent change.
- I can write an exponential function to represent a situation where there is repeated percent change.

Bridge

An article in the paper says that the local high school's student population will increase by 10% next year. Diego knows that this year, about 1,300 students attend the high school, and he wants to figure out next year's population. First, he draws this diagram.



Diego multiplies 1300 by 1.1 and gets 1430. Explain what he did and why this is correct.

Warm-up: Dandy Discounts

All books at a bookstore are 25% off. Priya bought a book originally priced at $32. The cashier applied the storewide discount and then took another 25% off for a coupon that Priya brought. If there was no sales tax, how much did Priya pay for the book? Show your reasoning.

Activity 1: Owing Interests

Tyler has a full tuition scholarship for their first year of college. To purchase books, they obtain a private student loan for \$450 that charges 18% interest. The loan is automatically deferred until 6 months after their final college semester, but it accumulates interest on the unpaid balance. Tyler makes no payments during the first year.

1. How much will they owe at the end of one year? Show your reasoning.

2. Assuming Tyler continues to make no payments to the lender, how much will they owe at the end of two years? Three years?

3. To find the amount owed at the end of the third year, Tyler wrote:

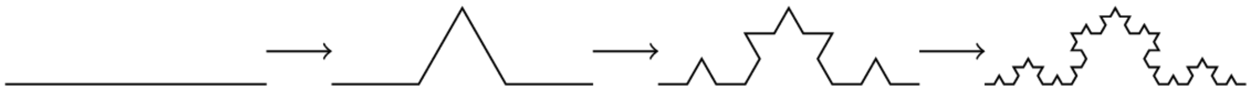
$$450 \cdot (1.18) \cdot (1.18) \cdot (1.18)$$

Does this expression correctly reflect the amount owed at the end of the third year? Explain or show your reasoning.

4. Write a function rule in the form $f(x) = ab^x$ for the amount Tyler owes at the end of x years without payment.

Are You Ready For More?

Start with a line segment of length 1 unit. Make a new shape by taking the middle third of the line segment and replacing it by two line segments of the same length to reconnect the two pieces. Repeat this process over and over, replacing the middle third of each of the remaining line segments with two segments each of the same length as the segment they replaced, as shown in the figure.



What is the perimeter of the figure after one iteration of this process (the second shape in the diagram)? After two iterations? After n iterations? Experiment with the value of your expression for large values of n.

Activity 2: Income from Movie Ticket Sales

The weekend a new movie was released, it had a total income of 270 thousand dollars. The weekly income for each week after initial release decreased by 13%.

1. The first week's income is what percentage of the income the weekend it was released?

2. What is the weekly income one week after initial release? Two weeks? Three weeks?

3. Write a function rule $f(x) =$ ______ for the weekly income after x weeks.

4. Explain what each number in the function rule means in this situation.

Lesson Debrief

Lesson 16 Summary and Glossary

Situations involving a repeated percent change can be represented using exponential expressions.

The following is an example of a repeated percent increase: When we borrow money from a lender, the lender usually charges *interest*, a percentage of the borrowed amount as payment for allowing us to use the money. The interest is usually calculated at a regular interval of time (monthly, yearly, etc.).

Suppose you received a loan of $500, and the interest rate is 15%, calculated at the end of each year.

- If you make no other purchases or payments, the amount owed after one year is 115% of $500. (100% is the amount of the loan plus the 15% in interest.)
- 115% is written in decimal form and used as the multiplier: $500 \cdot 1.15 = 575$.
- If you continue to make no payments or other purchases, in the second year, the amount owed would be 115% of $575.
- The table shows the calculation of the amount owed for the first 3 years.

Time in years	Amount owed in dollars
1	$500 \cdot (1.15)$
2	$500 \cdot (1.15)(1.15)$, or $500 \cdot (1.15)^2$
3	$500 \cdot (1.15)(1.15)(1.15)$, or $500 \cdot (1.15)^3$

- The pattern here continues. Each additional year means multiplication by another factor of (1.15). With no further purchases or payments, after t years, the debt in dollars is given by the expression: $500 \cdot (1.15)^t$

The following is an example of a repeated percent decrease: A company purchased new equipment at a cost of $100,000. The value of the equipment depreciates (decreases) at a rate of 12% each year.

- The value of the equipment after one year is 88% of $100,000. (100% of the original value minus 12% decrease in value.)
- 88% is written in decimal form and used as the multiplier: $100,000(0.88) = 88,000$.
- After the second year, the value of the equipment has decreased by 12% of the current value of $88,000.
- The table shows the calculation of the value of the equipment for the first 3 years.

Time in years	Value of equipment
1	$100,000 \cdot (0.88)$
2	$100,000 \cdot (0.88)(0.88)$, or $100,000 \cdot (0.88)^2$
3	$100,000 \cdot (0.88)(0.88)(0.88)$, or $100,000 \cdot (0.88)^3$

- The pattern here continues. Each additional year means multiplication by another factor of (0.88). After t years, the value of the equipment is given by the expression: $100,000 \cdot (0.88)^t$

Unit 6 Lesson 16 Practice Problems

1. In 2011, the population of deer in a forest was 650.

   a. In 2012, the population increases by 15%. Write an expression, using only multiplication, that represents the deer population in 2012.

   b. In 2013, the population increases again by 15%. Write an expression that represents the deer population in 2013.

   c. If the deer population continues to increase by 15% each year, write a function rule d that represents the deer population t years after 2011.

2. Mai and Elena are shopping for back-to-school clothes. They found a skirt that originally cost $30 on a 15% off sale rack. Today, the store is offering an additional 15% off. To find the new price of the skirt, in dollars, Mai says they need to calculate $30 \cdot 0.85 \cdot 0.85$. Elena says they can just multiply $30 \cdot 0.70$.

   a. How much will the skirt cost according to Mai's method?

   b. How much will the skirt cost according to Elena's method?

   c. Explain why the expressions used by Mai and Elena give different prices for the skirt. Which method is correct?

3. Automobiles start losing value, or depreciating, as soon as they leave the car dealership. Five years ago, a family purchased a new car that cost $16,490.

   If the car lost 13% of its value each year, what is the value of the car now?

4. The number of trees in a rainforest decreases each month by 0.5%. The forest currently has 2.5 billion trees.

   Write an expression to represent how many trees will be left in 10 years. Then evaluate the expression.

5. (*Technology required.*) One $1,000 loan charges 5% interest at the end of each year, while a second loan charges 8% interest at the end of each year.

***t*, number of years**	***b*, loan balance with 5% interest**	***c*, loan balance with 8% interest**
1		
2		
3		
t		

a. Complete the table with the balances for each loan. Assume that no payments are made and that the interest applies to the entire loan balance, including any previous interest charges.

b. Which loan balance grows more quickly? How will this be visible in the graphs of the two loan balances, b and c, as functions of the number of years, t?

c. Use technology to create graphs representing b and c over time. The graph should show the starting balance of each loan as well as the amount of the loan after 15 years. Write down the graphing window needed to show these points.

6. Lin opened a savings account that pays 5.25% interest annually and deposited $5,000.

If she makes no deposits and no withdrawals for 3 years, how much money will be in her account?

7. A person loans his friend \$500. They agree to an annual interest rate of 5%.

   Write a function rule for computing the amount owed on the loan, in dollars, after t years if no payments are made.

8. The real estate tax rate in 2018 in a small rural county is increasing by $\frac{1}{4}\%$. Last year, a family paid \$1,200.

   Which expression represents the real estate tax, in dollars, that the family will pay this year?

   a. $1,200 + 1,200 \cdot (\frac{1}{4})$

   b. $1,200 \cdot (1.25)$

   c. $1,200 \cdot (1.025)$

   d. $1,200 \cdot (1.0025)$

   (From Unit 6, Lessons 9 and 10)

9. Select **all** situations that are accurately described by the expression $15 \cdot 3^5$.

   a. A population of bacteria begins at 15,000. The population triples each hour. How many bacteria are there after 5 hours?

   b. A population of bacteria begins at 15,000. The population triples each hour. How many thousand bacteria are there after 5 hours?

   c. A population of bacteria begins at 15,000. The population quintuples each hour. How many thousand bacteria are there after 3 hours?

   d. A bank account balance is \$15. The account balance triples each year. What is the bank account balance, in dollars, after 5 years?

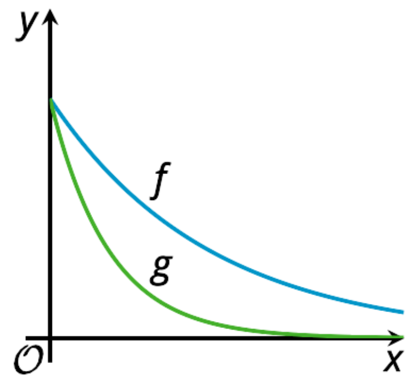
   e. A bank account balance is \$15,000. It grows by \$3,000 each year. What is the bank account balance, in thousands of dollars, after 5 years?

   (From Unit 6, Lesson 5)

10. Write the following expression with a single positive exponent: $(\frac{1}{2}y^{\frac{1}{3}})^{-4}$.

   (From Unit 6, Lessons 1 and 2)

11. Here are graphs of two exponential functions, f and g.



If $f(x) = 100 \cdot (\frac{2}{3})^x$ and $g(x) = 100 \cdot b^x$, what could be the value of b?

a. $\frac{1}{3}$

b. $\frac{3}{4}$

c. 1

d. $\frac{3}{2}$

(From Unit 5)

12. Jada and Priya have $20.00 each to spend at Students' Choice book store, where all students receive a 20% discount. They both want to purchase a copy of the same book, which normally sells for $22.50 plus 10% sales tax.[1]

- To check if she has enough to purchase the book, Jada takes 20% of $22.50 and subtracts that amount from the normal price. She takes 10% of the discounted selling price and adds it back to find the purchase amount.
- Priya takes 80% of the normal purchase price and then computes 110% of the reduced price.

Is Jada correct? Is Priya correct? Do they have enough money to purchase the book?

(Addressing NC.7.EE.3)

[1] Adapted from https://tasks.illustrativemathematics.org/

Lesson 17: Interpreting Rates

Learning Targets

- I can solve exponential equations in context by graphing.
- I can interpret the growth or decay rate given an exponential function in the form $f(x) = a \cdot b^x$.

Bridge

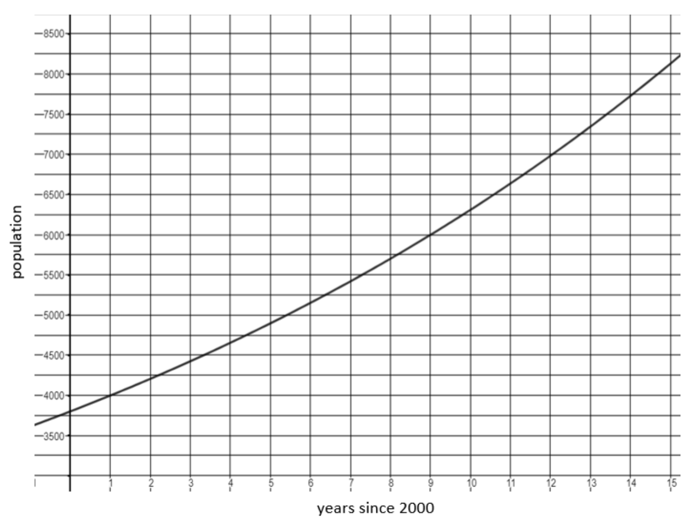
Express each percent change using an expression that only uses multiplication.

1. x increased by 5%
2. y decreased by 10%
3. z increased by 25%
4. w decreased by 2.5%

Warm-up: Small Town Population

The population in a small town can be represented by the function $p(t) = 3800(1.052)^t$, where t is the number of years since 2000.

1. What is the percent increase each year? Explain how you know.
2. Use the graph of the equation to find when the population was 7000 people.



Activity 1: Cancer Treatment

Iodine-131 is used to treat thyroid cancer. A typical amount used in therapy is 150 millicuries. (A millicurie is a unit used to measure radioactive materials.) Iodine-131 decays at a rate of 8.3% each day.

1. Write a function rule in the form $f(x) = ab^x$ to represent the amount of Iodine-131 in millicuries as a function of the number of weeks x.

2. Graph your function using graphing technology and use the graph to answer the following questions. Be prepared to show or explain your reasoning.

    a. After how many days will there be 100 millicuries of iodine-131?

    b. How many days does it take for there to be half of the original amount of iodine-131?

    c. When will there be less than 10 millicuries of iodine-131?

Are You Ready For More?

A patient received a second iodine-131 treatment 10 days after the first treatment. How many millicuries of iodine-131 would there be after 15 days since the first treatment?

Activity 2: What's the Rate?

Your teacher will give you a set of cards. Each person should select a card, determine whether the given function has a growth factor or a decay factor, and interpret what the growth or decay factor means in the situation. Take turns with your partner sharing your response.

As your partner shares, listen carefully to their interpretation. If you disagree, discuss your thinking and work to reach an agreement. Place the cards into two separate columns, one for growth and one for decay, and then choose another card.

When finished, arrange the column for growth in order from situations involving the least percent change to those with the greatest. Then arrange the column for decay in order from situations involving the least percent change to those with the greatest. List the percent changes in the table below.

Growth	Decay

Lesson Debrief

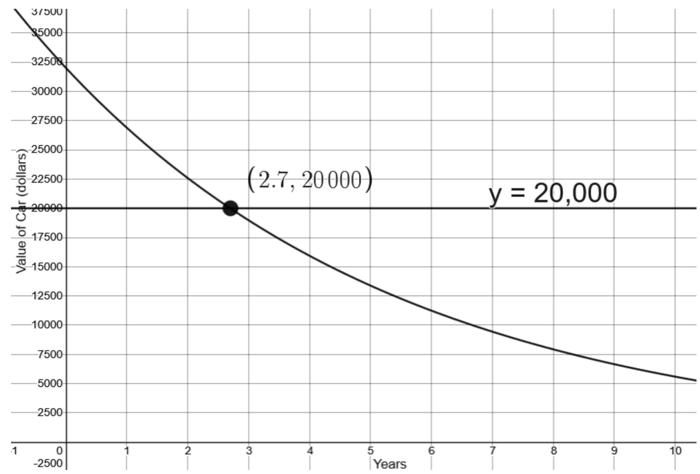
Lesson 17 Summary and Glossary

A car was purchased for $\$32,000$. The value of the car depreciates at a rate of 16% each year.

- A 16% decrease means that 84% of the value remains each year ($100\% - 16\% = 84\%$)
- The function rule $v(t) = 32,000(0.84)^t$ represents the value of the car as a function of time, t, in years since purchased.

We can use graphing to solve problems such as: "When will the value of the car be $20,000?"

- Graph the function.
- Graph the line $y = 20,000$.
- Identify the point where the horizontal line and the graph of the function intersect: $(2.7, 20,000)$
- The value of the car will be $20,000 approximately 2.7 years after being purchased.



Examining a function rule written in the form $f(x) = ab^x$ allows us to find the percent increase or decrease per unit of time.

The function $f(x) = 300(1.045)^x$ represents the account balance in dollars as a function of the number of years, x.

- The growth factor is 1.045, which is 104.5%.
- 100% + 4.5% = 104.5%.
- The account balance increases by a rate of 4.5% each year.

The function $b(t) = 264(0.86)^t$ represents the population of deer as a function of the number of years, t.

- The decay factor is 0.86, which is 86%.
- 100% - 14% = 86%.
- The population of deer decreases by 14% each year.

Unit 6 Lesson 17 Practice Problems

1. An algae bloom, if gone untreated, covers a lake at the rate of 2.5% each week. If it currently covers 13 square feet, how many weeks will it take to cover 100 square feet?

2. A computer valued at $1,200 depreciates at a rate of 23% each year. After how long will the computer be approximately valued at $250?

3. Mai used a computer simulation to roll number cubes and count how many rolls it took before all of the cubes came up sixes. Here is a table showing her results.

d, number of cubes	1	2	3	4
r, number of rolls	5	31	143	788

Would a linear or exponential function be appropriate for modeling the relationship between d and r? Explain how you know.

(From Unit 6, Lesson 13)

4. The table shows the height of a ball after different numbers of bounces.

n	1	2	3	4	5
h	83	61	46	35	26

a. Can the height, h, in centimeters, after n bounces be modeled accurately by a linear function? Explain your reasoning.

b. Can the height, h, after n bounces be modeled accurately by an exponential function? Explain your reasoning.

c. Create a model for the height of the ball after n bounces and plot the predicted values with the data.

d. Use your model to estimate the height the ball was dropped from.

e. Use your model to estimate how many bounces it takes before the rebound height is less than 10 cm.

(From Unit 6, Lesson 13)

5. Match each function rule with its corresponding percent change.

Function	Percent change
a. $f(x) = 200(0.75)^x$	1. 3% decrease
b. $g(x) = 50(1.25)^x$	2. 25% increase
c. $h(x) = 25(3)^x$	3. 25% decrease
d. $j(x) = 75(0.97)^x$	4. 200% increase

6.

a. Give a positive value for x, such that $x^4 > x^{-4}$.

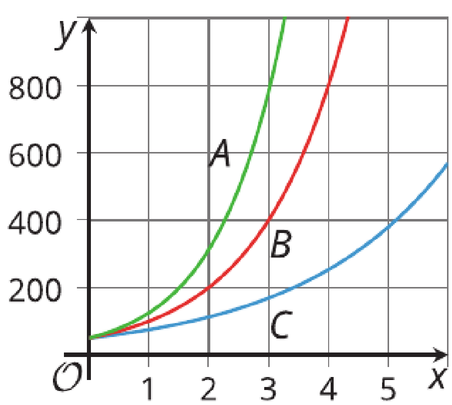
b. Give a positive value for x, such that $x^4 < x^{-4}$.

c. Give a positive value for x, such that $x^4 = x^{-4}$.

(From Unit 6, Lessons 1 and 2)

7. Here are the graphs of three equations: $y = 50 \cdot (1.5)^x$, $y = 50 \cdot 2^x$, and $y = 50 \cdot (2.5)^x$

Which equation matches each graph? Explain how you know.



(From Unit 5)

8. The bungee jump in Rishikesh, India is 83 meters high. The jumper free falls for 5 seconds to about 30 meters above the river.

   a. Draw a graph of the bungee jump in Rishikesh.

   b. Identify and describe three pieces of important information you can learn from the graph of the bungee jump.

(From Unit 5)

9. A major retailer has a staff of 6,400 employees for the holidays. After the holidays, they will decrease their staff by 30%.

   How many employees will they have after the holidays?

(Addressing NC.7.RP.3)

Lesson 18: Which One Changes Faster?

Learning Targets

- I can use tables, calculations, and graphs to compare growth rates of linear and exponential functions and predict how the quantities change eventually.

Bridge

You are a representative for a cell phone company, and it is your job to promote different texting plans. Functions a and b give the cost of each plan, where t is the number of text messages sent.[1]

Plan A: $a(t) = 29.95 + 0.10t$

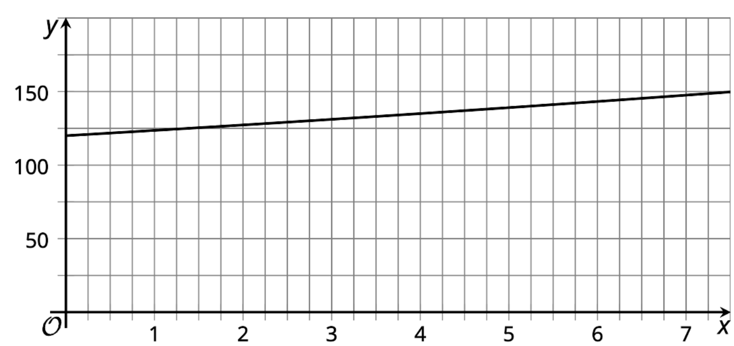
Plan B: $b(t) = 49.94 + 0.05t$

1. Which plan is the better deal for someone who usually sends around 100 texts per month?

2. Do you think this plan is best for everyone? Explain your reasoning.

Warm-up: Graph of Which Function?

Here is a graph.



1. Which equation do you think the graph represents, equation “a” or “b”? Use the graph to support and explain your reasoning.

   a. $y = 120 + (3.7) \cdot x$

   b. $y = 120 \cdot (1.03)^x$

2. What information might help you decide more easily whether the graph represents a linear or an exponential function?

[1] Adapted from https://tasks.illustrativemathematics.org/

Activity 1: Simple and Compound Interests

A family has $1,000 to invest and is considering two options: investing in government bonds that offer 2% simple interest or investing in a savings account at a bank, which charges a $20 fee to open an account and pays 2% compound interest. Both options pay interest annually.

Here are two tables showing what they would earn in the first couple of years if they do not invest additional amounts or withdraw any money.

Bonds

Years of investment	Amount in dollars
0	$1,000
1	$1,020
2	$1,040

Savings account

Years of investment	Amount in dollars
0	$980
1	$999.60
2	$1,019.59

1. Describe the way the investment in bonds grows with simple interest.

2. For the savings account, how are the amounts $999.60 and $1,019.59 calculated?

3. For each option, write an equation to represent the relationship between the amount of money and the number of years of investment.

4. Which investment option should the family choose? Use your equations or calculations to support your answer.

5. Use graphing technology to graph the two investment options and show how the money grows in each.

Activity 2: Reaching 2,000

1. $f(x) = 2x$ and $g(x) = (1.01)^x$

   Complete the table of values for the functions f and g.

x	$f(x)$	$g(x)$
1		
10		
50		
100		
500		

2. Based on the table of values, which function do you think grows faster? Explain your reasoning.

3. Which function do you think will reach a value of 2,000 first? Show your reasoning.

Are You Ready For More?

Consider the functions $g(x) = x^5$ and $f(x) = 5^x$. While it is true that $f(7) > g(7)$, for example, it is hard to check this using mental math. Find a value of x for which properties of exponents allow you to conclude that $f(x) > g(x)$ without a calculator.

Lesson Debrief

Lesson 18 Summary and Glossary

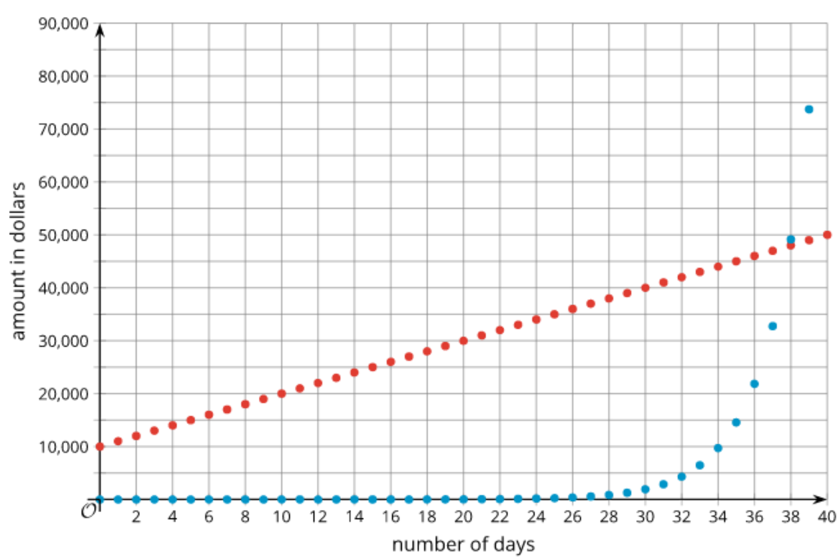
Suppose that you won the top prize from a game show and are given two options. The first option is a cash gift of $\$10,000$ and $\$1,000$ per day for the next 7 days. The second option is a cash gift of 1 cent (or $\$0.01$) that grows tenfold each day for 7 days. Which option would you choose?

In the first option, the amount of money increases by the same amount ($\$1,000$) each day, so we can represent it with a linear function. In the second option, the money grows by multiples of 10, so we can represent it with an exponential function. Let f represent the amount of money x days after winning with the first option and let g represent the amount of money x days after winning with the second option.

f	$f(x)$	$g(x)$
x	$10,000 + 1,000x$	$(0.01) \cdot 10^x$
1	$11,000$	0.1
2	$12,000$	1
3	$13,000$	10
...	...	...
6	$16,000$	$10,000$
7	$17,000$	$100,000$

For the first few days, the second option trails far behind the first. Because of the repeated multiplication by 10, however, after 7 days it surges past the amount in the first option.

What if the growth factor factor of growth is much smaller than 10? Suppose we have a third option, represented by a function h. The starting amount is still $\$0.01$ and it grows by a factor of 1.5 times each day. In this If we graph of the function $h(x) = (0.01) \cdot (1.5)^x$, along with the linear function $f(x) = 10,000 + 1,000x$, we see that it takes many, many more days before we see a rapid growth. But given time to continue growing, the amount in this exponential option will eventually also outpace that in the linear option. When we say that exponential functions will eventually grow faster than linear functions, we are comparing their end behavior.



Unit 6 Lesson 18 Practice Problems

1. Functions a, b, c, d, e, and f are given below. Classify each function as linear, exponential, or neither.

   a. $a(x) = 3x$

   b. $b(x) = 3^x$

   c. $c(x) = x^3$

   d. $d(x) = 9 + 3x$

   e. $e(x) = 9 \cdot 3^x$

   f. $f(x) = 9 \cdot 3x$

2. Here are four equations defining four different functions, a, b, c, and d. List them in order of increasing rate of change. That is, start with the one that grows the slowest and end with the one that grows the quickest.

   - $a(x) = 5x + 3$
   - $b(x) = 3x + 5$
   - $c(x) = x + 4$
   - $d(x) = 1 + 4x$

3. (*Technology required.*) Function f is defined by $f(x) = 3x + 5$ and function g is defined by $g(x) = (1.1)^x$.

   a. Complete the table with values of $f(x)$ and $g(x)$. When necessary, round to 2 decimal places.

x	$f(x)$	$g(x)$
1		
5		
10		
20		

   b. Which function do you think grows faster? Explain your reasoning.

   c. Use technology to create graphs representing f and g. What graphing window do you have to use to see the value of x where g becomes greater than f for that x?

4. Functions m and n are given by $m(x) = (1.05)^x$ and $n(x) = \frac{5}{8}x$. As x increases from 0:

   a. Which function reaches 30 first?

   b. Which function reaches 100 first?

5. The functions f and g are defined by $f(x) = 8x + 33$ and $g(x) = 2 \cdot (1.2)^x$.

    a. Which function eventually grows faster, f or g? Explain how you know.

    b. Explain why the graphs of f and g meet for a positive value of x.

6. For each function rule, identify the percent increase or percent decrease.

    a. $f(x) = 11(1.34)^x$

    b. $g(x) = 86(0.82)^x$

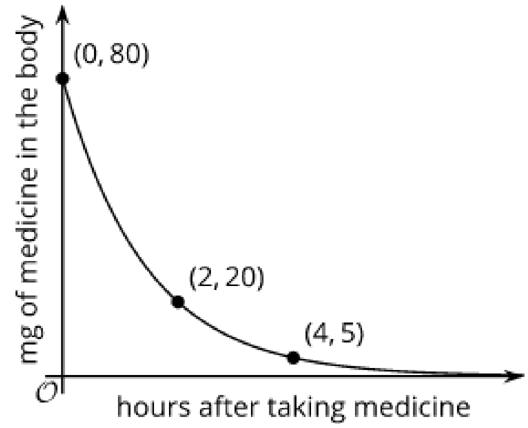
(From Unit 6, Lesson 17)

7. The average price of a gallon of regular gasoline in 2016 was $2.14. In 2017, the average price was $2.42 a gallon—an increase of 13%.

    At that rate, what will the average price of gasoline be in 2020?

(From Unit 6, Lesson 16)

8. The function f represents the amount of a medicine, in mg, in a person's body t hours after taking the medicine. Here is a graph of f.



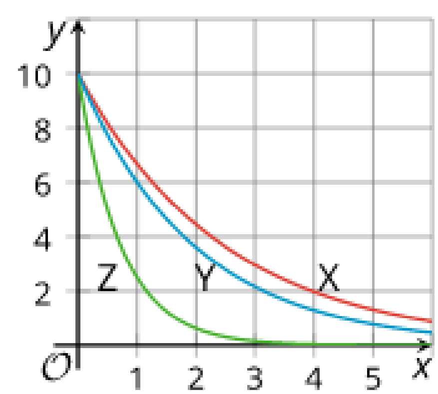
a. How many mg of the medicine did the person take?

b. Write an equation that defines f.

c. After 7 hours, how many mg of medicine remain in the person's body?

(From Unit 6, Lesson 15)

9. Here are the graphs of three functions. Which of these functions decays the most quickly? Which one decays the least quickly?



(From Unit 6, Lesson 14)

10. A piece of paper is 0.004 inches thick.

    a. Explain why the thickness in inches, t, is a function of the number of times the paper is folded, n.

    b. Using function notation, represent the relationship between t and n. That is, find a function f so that $t = f(n)$.

(From Unit 6, Lesson 11)

11. For each of the functions f, g, h, p, and q, the domain is $0 \leq x \leq 100$.

    For which functions is the average rate of change a good measure of how the function changes for this domain? Select **all** that apply.

    a. $f(x) = x + 2$

    b. $g(x) = 2^x$

    c. $h(x) = 111x - 23$

    d. $p(x) = 50,000 \cdot 3^x$

    e. $q(x) = 87.5$

(From Unit 5)

Lessons 19 & 20: Mathematical Modeling[1]

Learning Targets

- I can use mathematics to model real-world situations.
- I can test and improve mathematical models for accuracy in representing and predicting real things.

Advice on Modeling

These are some steps that successful modelers often take and questions that they ask themselves. You don't necessarily have to do all of these steps, or do them in order. Only do the parts that you think will help you make progress.

Understand the Question
Think about what the question means before you start making a strategy to answer it. Are there words you want to look up? Does the scenario make sense? Is there anything you want to get clearer on before you start? Ask your classmates or teacher if you need to.

Refine the Question
If necessary, rewrite the question you are trying to answer so that it is more specific.

Estimate a Reasonable Answer
If you don't have enough information to decide what's reasonable, try to come up with an answer that would be too low, and an answer that would be too high.

Identify Unknowns
- What are the meaningful quantities in this situation? Write them down.
- What information would be useful to know? In order to get that information, you could: look it up, take a measurement, or make an assumption.

Gather Information
Write down any of the unknown information that you find. Organize your information in a way that makes sense to you.

Experiment!
Try different ideas to make progress toward answering your question. If you are stuck, think about:
- Helpful ways to organize the information you have or organize your work
- Questions you *can* answer using the information you have
- Ways to represent mathematical relationships or sets of data (tables, equations, graphs, statistical plots)
- Tools that are available for representing mathematics, both digital and analog

Check Your Reasoning
Do you have a first answer to your question? Great! See if it's reasonable.
- Make sure you can explain what the answer means in terms of the original problem.
- Check your precision: Is your answer overly precise? Not precise enough?

Use and Improve Your Model
- Did you make assumptions or measurements? How can you express your model more generally, so that it would work for a range of numbers instead of the specific numbers you used?
- What are the limitations of your model? That is, what are some ways it is not realistic? Does it only work for certain inputs but not others? Are there any meaningful inputs affecting the outcome that are not accounted for? If possible, improve your model to take these into account.
- What are the implications of your model? That is, what should people or organizations do differently or smarter as a result of what your model shows? What would be effective ways to communicate with them?
- What are the areas for further research? That is, what new things are you wondering about that could be investigated, by you or someone else?

[1] Adapted from IM 9–12 Math Algebra 1 Modeling Prompts https://curriculum.illustrativemathematics.org/HS/index.html, copyright 2019 by Illustrative Mathematics. Licensed under the Creative Commons Attribution 4.0 license https://creativecommons.org/licenses/by/4.0/.

Modeling Rubric

<table>
<tr><th rowspan="2">Skill</th><th colspan="3">Score</th><th rowspan="2">Notes or Comments</th></tr>
<tr><th>Proficient</th><th>Developing</th><th>Needs Revisiting</th></tr>
<tr><td rowspan="2">1. Decide What to Model</td><td>● Assumptions made are clearly identified and justified. Resulting limitations are stated when appropriate.
● Variables of interest are clearly identified and chosen wisely, and appropriate units of measure are used.</td><td>● Assumptions are noted but lacking in justification or difficult to find.
● Variables of interest are noted, but may lack justification, be difficult to find, or not be measured with appropriate units.</td><td>● No assumptions are stated.
● No variables are defined.</td><td></td></tr>
<tr><td colspan="4">To improve at this skill, you could:
● Ask questions about the situation to understand it better
● Check the assumptions you're making to see if they're reasonable (Try asking a friend, or imagining that you're a person involved in the scenario. Would those assumptions make sense to you?)
● Double-check the variables you've identified: Are there other quantities in the situation that could vary? Is there something you've identified as a variable that is actually fixed or determined? (Remember that more abstract things like time and speed are also quantities.)</td></tr>
<tr><td rowspan="2">2. Formulate a Mathematical Model</td><td>● An appropriate model is chosen and represented clearly.
● Diagrams, graphs, etc. are clear and appropriately labeled.</td><td>Parts of the model are unclear, incomplete, or contain mistakes.</td><td>No model is presented, or the presentation contains significant errors.</td><td></td></tr>
<tr><td colspan="4">To improve at this skill, you could:
● Check your model more carefully to make sure it really fits well
● Consider a wider variety of possible models, to find one that fits the situation better
● Think about the situation more deeply before trying to find a model
● Convince a skeptic: Pretend that you think your model is inadequate, or ask a friend to pretend to be skeptical of it. What would a skeptic find wrong with your model? Try to fix those things, or explain why they're not actually problems.</td></tr>
</table>

<table>
<tr><th rowspan="2">Skill</th><th colspan="3">Score</th><th rowspan="2">Notes or Comments</th></tr>
<tr><th>Proficient</th><th>Developing</th><th>Needs Revisiting</th></tr>
<tr><td rowspan="2">3. Use Your Model to Reach a Conclusion</td><td>● Solution is relevant to the original problem.
● Reader can easily understand the reasoning leading to the solution.
● Relevant details are included like units of measure.</td><td>Solution is not well-aligned to the original problem, or aspects of the solution are difficult to understand or incomplete.</td><td>No solution is provided.</td><td></td></tr>
<tr><td colspan="4">To improve at this skill, you could:
● Double-check your calculations: Show them to someone else to see if they agree, or take a break and look at your calculations again later
● Make sure your calculations are justified by your model: Ask yourself how you decided what to calculate, and see if your reasoning matches up with your model
● Think more deeply about what your conclusions mean in the original scenario: Imagine you're a person involved in the scenario, or explain your conclusions to someone else and see if they have questions</td></tr>
<tr><td rowspan="2">4. Refine and Share Your Model</td><td>● The model's implications are clearly stated.
● The limitations of the model and solution are addressed.</td><td>The limitations of the model and solution are addressed but lacking in depth or ignoring key components.</td><td>No interpretation of model and solution is provided.</td><td></td></tr>
<tr><td colspan="4">To improve at this skill, you could:
● Think more creatively about what your conclusions mean: Ask yourself "If I was involved in this situation, what would I understand better because of these conclusions? What would I want to do next?"
● Be skeptical of your model: What don't you like about it, and what can you do to fix those things?
● Explain your model to someone else: Tell them how it works and why it's good. If you're not sure how it works or why it's good, you might need to change it.</td></tr>
</table>

Workspace for Modeling Prompt # _____

Workspace for Modeling Prompt # _____

Modeling Prompt # ____ Reflection

Lesson 21: Post-Test Activities

Learning Targets

- I can determine whether to use a linear function or an exponential function to model real-world data.
- I can determine how well a chosen model fits the given information.

Activity 2: Population Predictions

Here are population data for three cities at different times between 1950 and 2000. The following questions and prompts will help you to decide what the data tells us, if anything, about the current population in the cities or what the population will be in 2050?[1]

City	1950	1960	1970	1980	1990	2000
Paris	6,300,000	7,400,000	8,200,000	8,700,000	9,300,000	9,700,000
Austin	132,000	187,000	254,000	346,000	466,000	657,000
Chicago	3,600,000	3,550,000	3,400,000	3,000,000	2,800,000	2,900,000

1. What do you notice? What do you wonder? Take a minute to think individually and then share with your group.

2. Each student in the group should select a different city. How would you describe the population change in each city during this time period? Write one to two sentences, then discuss with your group.

3. Use technology to graph the population data for each city, with each student creating a graph for their selected city and sharing with the group.

[1] Adapted from IM 9–12 Math Algebra 1, Unit 5, Lesson 21 https://curriculum.illustrativemathematics.org/HS/teachers/index.html,

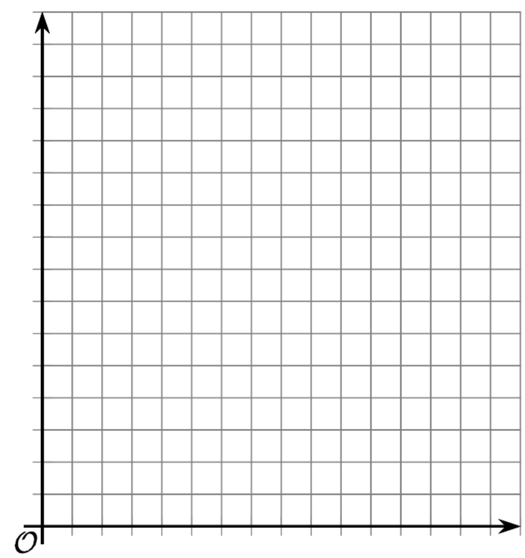
4. What kind of model (linear, exponential, both, or neither) do you think is appropriate for each city population? Each group member should make a choice for their selected city and explain their reasoning to the group. Use the sentence frames to engage in discussion:

    - A ____ model is appropriate to model the population data for ______ because _____.
    - I agree a ____ model is appropriate to model the population data for ____ because ___ and also because ___.
    - I disagree that ___model is appropriate to model the population data for ___ because ___.

5. Working together with your group members, for each population that you agreed can be modeled by a linear and or exponential function:

    a. Write an equation for the function(s).

    b. Graph the function(s) and the data on the same coordinate plane.

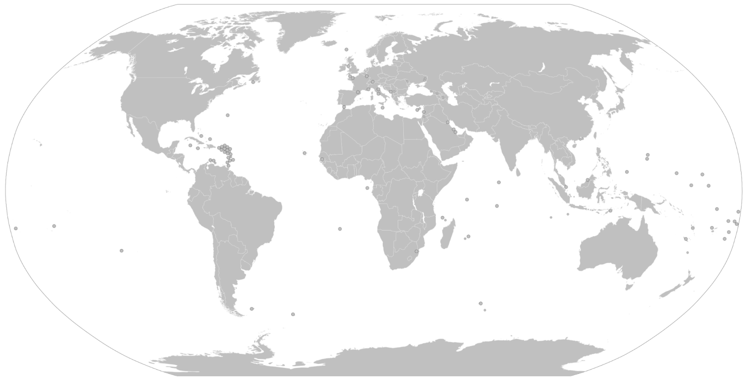


6. Working together with your group members, compare the graphs of your functions with the actual population data to determine how well the models fit the data.

    a. Use your models to predict the population in each city in 2010, the current year, and 2050 (three predictions).

    b. Do you think that these predictions are (or will be) accurate? Explain your reasoning.

Now you will explore world population data. If you would like to include additional data points, use the first two columns of the historical data table found at: https://bit.ly/PopulationWorld. All of these questions should be completed collaboratively with your group.



(Image[2])

Year	1804	1927	1960	1974	1987	1999	2011	2020
World population in billions	1	2	3	4	5	6	7	7.8

7. Use technology to graph the world population data. Make sure you adjust the window to fit all of your data.

8. Take a minute to think individually about whether or not a linear function would be appropriate for modeling the world population growth over the last 200 years. Use the sentence frames to engage in discussion:

    - A linear model (would/would not) be appropriate because ___.
    - I agree a linear model (is/is not) appropriate to model the world population data for the last 200 years because ___ and also because ___.
    - I disagree that a linear model (is/is not) appropriate to model the world population data for because ___.

9. If you agree that a linear model is appropriate for the last 200 years, find a linear model using technology.

[2] https://commons.wikimedia.org/wiki/Maps_of_the_world#/media/File:BlankMap-World-with-Circles.svg.

10. Take a minute to think individually about whether or not an exponential function would be appropriate for modeling the world population growth over the last 200 years. Use the sentence frames to engage in discussion:

    - An exponential model <u>(would/would not)</u> be appropriate because ___.
    - I agree an exponential model <u>(is/is not)</u> appropriate to model the world population data for the last 200 years because ___ and also because ___.
    - I disagree that an exponential model <u>(is/is not)</u> appropriate to model the world population data for because ___.

11. If you agree that an exponential model is appropriate for the last 200 years, find an exponential model using technology.

12. If the growth trend continues, use one or both of your models to predict what the world population will be in 2050. Do you think the prediction(s) are reasonable? Explain your reasoning.

13. Do you think there is a limit to how long the model will provide accurate predictions? Why or why not?